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Josephson-Vortex Bloch Oscillations: Single-Pair Tunnelling Effect.

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Abstract. – We consider the Josephson-vortex motion in a long one-dimensional Josephson junction in a thin film. We show that this Josephson vortex is similar to a mesoscopic capacitor. We demonstrate that a single-Cooper-pair tunnelling results in non-linear Bloch-type oscillations of a Josephson vortex in a current-biased Josephson junction. We find the frequency and the amplitude of this motion.

Single electron and Cooper-pair tunnelling remain a fundamental problem in normal metal and superconductivity physics. Under detailed experimental and theoretical study is electron transport through tunnel junctions in mesoscopic systems [1, 2]. A particular attention has been given also to the classical and quantum dynamics of Josephson vortices in Josephson junctions and arrays of Josephson junctions. Very fast classical motion of Josephson vortices with velocities approaching the Swihart velocity was treated experimentally in annular Josephson tunnel junctions [3, 4]. The ballistic motion of vortices in two-dimensional arrays of Josephson junctions is under thorough theoretical and experimental study [5, 6]. It was shown that the spin waves excitation results in an intrinsic friction when a vortex is moving in an array of Josephson junctions [6-10]. Vortices tunnelling is treated theoretically and experimentally in two-dimensional arrays of Josephson junctions [11, 12, 6].

In this letter we consider the effect of a quantum single-Cooper-pair tunnelling on the classical motion of a Josephson vortex in a long Josephson junction. We point out that a Josephson vortex in a thin film is similar to a mesoscopic capacitor. We show that at sufficiently low temperatures it results in non-linear Bloch-type oscillations in a current-biased Josephson junction. We find the amplitude and frequency of this motion.

To develop the analogy between a Josephson vortex and a mesoscopic capacitor let us consider a superconducting film with a Josephson junction along the x-axis (see fig. 1). The motion of a Josephson vortex along this Josephson junction induces voltage $V(x, t)$. The value of the voltage drop across the Josephson junction is equal to [13]

$$V = \frac{\hbar}{2e}\frac{\partial \varphi}{\partial t}, \tag{1}$$

where $\varphi(x, t)$ is the phase difference. This voltage results in a certain charge Q moving simultaneously with the Josephson vortex. As $\varphi(x, t) = \varphi(x - vt)$ and $\varphi(+\infty) - \varphi(-\infty) = 2\pi$, the value of Q takes the explicit form

$$Q = \int_{-\infty}^{\infty} \rho(x - vt)\,\mathrm{d}x = \frac{\hbar}{2e} dC \int_{-\infty}^{\infty} \frac{\partial \varphi}{\partial t}\,\mathrm{d}x = \frac{\pi\hbar}{e} dCv, \tag{2}$$

where d is the film thickness, C is the specific capacity of the Josephson junction, and v is the Josephson-vortex velocity. The charge density $\rho(x - vt)$ is distributed in the cross-section of the Josephson junction over an area of the order of dl, where l is the characteristic space scale of the phase difference distribution, *i.e.* the size of the Josephson vortex.

Thus, a moving Josephson vortex is similar to a charged mesoscopic capacitor with a certain effective capacity C_{eff}. The value of C_{eff} is proportional to the area dl, where the charge Q is localized. Mesoscopic effects affect the motion of a Josephson vortex if the effective capacity $C_{\text{eff}} \propto dl$ is small, *i.e.* in the case of small d and l.

Let us now consider a Josephson vortex in a thin film meaning that $d \ll \lambda$, where λ is the London penetration depth. The space and time dependences of the phase difference $\varphi(x, t)$ determine the statics and dynamics of a Josephson vortex. To find the dependence $\varphi(x, t)$ we use the continuity equation $\operatorname{div} \boldsymbol{j} = 0$, where $\boldsymbol{j}$ is the current density. We treat here the case of zero dissipation, *i.e.* the case of an ideal Josephson junction and calculate the current density taking into account the superconducting, tunnelling and displacement currents. We neglect the resistive (leakage and quasi-particle tunnelling) currents as they can be minimized by the Josephson- junction design and by lowering the temperature. An additional friction force arises as a result of Josephson-vortex Cherenkov radiation of small-amplitude electromagnetic waves propagating along the tunnel junction [14]. The contribution of this radiation friction force becomes important only when the Josephson-vortex velocity is approaching the Swihart velocity. Note that the dissipation due to Josephson-vortex Cherenkov radiation in a long Josephson junction is similar to the dissipation due to spin waves excitation in Josephson-junction arrays [6-10].

In the thin-film limit the density of the superconducting current and magnetic field in the superconductor decrease with the characteristic space scale [15]

$$\lambda_{\text{eff}} = \frac{\lambda^2}{d} \gg \lambda. \tag{3}$$

To emphasize the mesoscopic effects we consider the case of a small-size Josephson vortex, *i.e.* we assume that $l \ll \lambda_{\text{eff}}$. Then the relation between the superconducting current

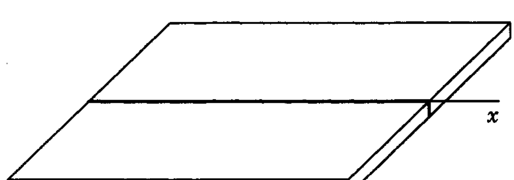

Fig. 1. – A superconducting film with a Josephson junction (thick line).

density and the phase difference becomes non-local and the continuity equation leads to a non-linear integro-differential equation determining the value of $\varphi(x, t)$ [16,17]. In particular, in the case of $d \ll \lambda$ and $l \ll \lambda_{\text{eff}}$ this equation reads [18]

$$\sin\varphi + \frac{1}{\omega_{\text{p}}^2}\frac{\partial^2\varphi}{\partial t^2} = \frac{l_{\text{J}}}{\pi}\int_{-\infty}^{\infty}\frac{\text{d}y'}{y'-y}\frac{\partial\varphi}{\partial y'}\,, \tag{4}$$

where

$$\omega_{\text{p}} = \left(\frac{2ej_{\text{c}}}{C\hbar}\right)^{1/2} \tag{5}$$

is the plasma frequency,

$$l_{\text{J}} = \frac{c\Phi_0}{16\pi^2\lambda^2 j_{\text{c}}}\,, \tag{6}$$

Φ_0 is the flux quantum, and j_{c} is the Josephson-critical-current density.

Let us first consider a stationary Josephson vortex. In this case $v = 0$ and the space distribution of the phase difference $\varphi(x)$ is given by the exact solution of eq. (4) [16,17]

$$\varphi(x) = 2\,\text{arctg}\left(\frac{x}{l_{\text{J}}}\right) + \pi\,. \tag{7}$$

If the velocity v of a Josephson vortex is small, *i.e.* $v \ll \omega_{\text{p}} l_{\text{J}}$, the solution of eq. (4) is given by eq. (7), where instead of x we substitute $x - vt$. In particular, it follows from eq. (7) that the size of a Josephson vortex moving with a velocity $v \ll \omega_{\text{p}} l_{\text{J}}$ is of the order of l_{J}.

The motion of a Josephson vortex induces an electrical field localized inside the Josephson junction. Using eq. (1) we find that the energy of this electrical field is equal to

$$\mathscr{E}_{\text{v}} = \frac{\hbar^2 Cd}{8e^2}\int_{-\infty}^{\infty}\left(\frac{\partial\varphi}{\partial t}\right)^2 dx = \frac{\hbar^2 C dv^2}{8e^2}\int_{-\infty}^{\infty}\left(\frac{\partial\varphi}{\partial x}\right)^2 \text{d}x\,. \tag{8}$$

It follows from eq. (8) that $\mathscr{E}_{\text{v}} \propto v^2$. Combining eqs. (8) and (2) we find that at the same time $\mathscr{E}_{\text{v}} \propto Q^2$. We can thus introduce the Josephson-vortex mass M and effective capacity C_{eff} presenting $\mathscr{E}_{\text{v}}$ as

$$\mathscr{E}_{\text{v}} = \frac{Mv^2}{2} = \frac{Q^2}{2C_{\text{eff}}}\,, \tag{9}$$

where

$$M = \frac{\pi\hbar^2 Cd}{2e^2 l_{\text{J}}}\,, \tag{10}$$

and

$$C_{\text{eff}} = 2\pi Cdl_{\text{J}}\,. \tag{11}$$

Let us now consider the motion of a Josephson vortex along a Josephson junction. We treat the case when the driving force is the Lorentz force F_{L} resulting from a bias current across the Josephson junction. Then the equation of motion reads

$$M\frac{dv}{dt} = \frac{\Phi_0 jd}{c}, \tag{12}$$

where j is the bias current density. Using eq. (2) we rewrite eq. (13) as

$$\frac{dQ}{dt} = 2\pi jdl_J. \tag{13}$$

It follows from eqs. (12) and (13) that the bias current results in the Josephson-vortex acceleration and charging.

Let us now treat a single Cooper pair crossing a Josephson junction. This elementary recharging process ($Q \to Q - 2e$) is changing the electrical-field energy of a Josephson vortex by $\Delta\mathcal{E}_v = \mathcal{E}_v(Q-2e) - \mathcal{E}_v(Q)$. A single-Cooper-pair tunnelling can happen if the energy difference $\Delta\mathcal{E}_v$ is equal to zero. It follows from eq. (9) that $\Delta\mathcal{E}_v = 0$ when the charge $Q = e$. The value of Q is equal to the electron charge e at a certain Josephson-vortex velocity v_m. It follows from eq. (2) that

$$v_m = \frac{e^2}{\pi\hbar}\frac{1}{Cd}. \tag{14}$$

This reasoning is true if the elementary charging energy of a Josephson vortex $\mathcal{E}_v(e)$ is larger than the scale of the thermal fluctuations $k_B T$, *i.e.* if

$$\frac{e^2}{2C_{\text{eff}}} = \frac{e^2}{4\pi Cdl_J} \gg k_B T, \tag{15}$$

where T is the temperature and k_B is the Boltzmann constant. Note that the inequality given by eq. (15) restricts the film thickness to $d \ll e^2/4\pi k_B TCl_J$.

Thus, a single-Cooper-pair tunnelling is changing the charge of a moving Josephson vortex from e to $-e$. It follows from eq. (2) that simultaneously this tunnelling event is changing the velocity of a Josephson vortex from v_m to $-v_m$. It means that in the mainframe of a classical mechanics approach to the motion of a Josephson vortex, a single-Cooper-pair tunnelling reveals as an elastic impact. The effect of this impact is as follows. The Lorentz force F_L is increasing the velocity of a Josephson vortex v with a constant acceleration $a_v = F_L/M$. At the moment when the value of v becomes equal to v_m an elastic impact (a single-Cooper-pair tunnelling event) happens changing v_m to $-v_m$. As the velocity of a Josephson vortex continues to increase with the rate given by a_v, the process repeats itself periodically. It

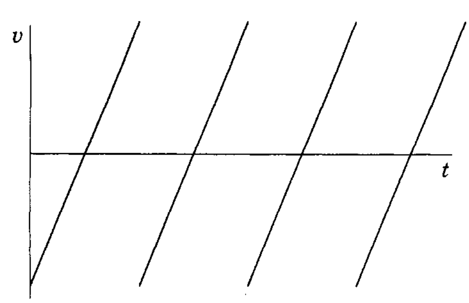

Fig. 2. – The dependence of the Josephson-vortex velocity on time (arbitrary units).

follows from eqs. (12) and (13) that the frequency ω and the amplitude l_m of this periodic Josephson-vortex motion are given by

$$\omega = \frac{2\pi^2 jdl_J}{e}, \tag{16}$$

$$l_m = \frac{e^3}{4\pi^2 \hbar C d^2 j\, l_J}. \tag{17}$$

The dependence of the Josephson-vortex velocity on time is shown in fig. 2.

To estimate the values of ω and l_m we substitute in eq. (17) the expression for l_J given by eq. (6) and the specific capacity C in the form

$$C = \frac{\varepsilon}{4\pi d_0}, \tag{18}$$

where ε is the dielectric constant and d_0 is the distance between the banks of the Josephson junction. It results in the formulae

$$\omega = \frac{\pi}{8}\frac{\hbar c}{e^2}\frac{j}{j_c}\frac{cd}{\lambda^2}, \quad \text{and} \quad l_m = \frac{16}{\varepsilon}\left(\frac{e^2}{\hbar c}\right)^2\left(\frac{\lambda}{d}\right)^2\frac{j_c}{j}\,d_0. \tag{19}$$

Using the data $\lambda \approx 3\cdot 10^{-5}$ cm, $d \approx 10^{-6}$ cm, $d_0 \approx 10^{-8}$ cm, and $\varepsilon \approx 5$, we find an estimation for the values of ω and l_m in the form

$$\omega \approx 2\cdot 10^{15}\frac{j}{j_c}\,\text{s}^{-1}, \quad \text{and} \quad l_m \approx 1.5\cdot 10^{-9}\frac{j_c}{j}\,\text{cm}. \tag{20}$$

It follows from eqs. (19) and (20) that if $j/j_c \approx 10^{-8}$, then $\omega \approx 2\cdot 10^7\,\text{s}^{-1}$ and $l_m \approx 0.15$ cm. Note that for the same data as above and the critical current density $j_c \approx 10^5\,\text{A/cm}^2$, we estimate $l_J \approx 1.5\cdot 10^{-4}$ cm, $C_{\text{eff}} \approx 7.5\cdot 10^{-3}$ cm and the characteristic temperature

$$\frac{e^2}{2C_{\text{eff}}k_B} \approx 0.1\,\text{K}. \tag{21}$$

We present in this letter a semi-classical approach to the mesoscopic aspects of Josephson-vortex dynamics in a thin film. Namely, in the above consideration we decouple the tunnelling current forming the Josephson vortex and the process of Josephson-vortex recharging. To verify this approach, let us introduce two characteristic frequencies.

The first frequency, ω_c, is typical for the motion of Josephson-vortex current carriers. We introduce it as

$$\omega_c = \frac{j_c}{nel_J}, \tag{22}$$

where n is the density of the electrons.

The second frequency, ω_e, is characteristic for the tunnelling process of charge transfer through the Josephson junction. We introduce it as

$$\omega_e = \frac{j_c l_J d}{e}. \tag{23}$$

The ratio of these two frequencies is proportional to the density of electrons and in all cases of practical interest is very high, *i.e.*

$$\frac{\omega_e}{\omega_c} = ndl_J^2 \gg 1 . \tag{24}$$

The strong inequality given by eq. (24) is the physical reason to ignore the coupling between the intrinsic current pattern and the mesoscopic aspects of Josephson-vortex motion.

To summarize, we show that a Josephson vortex in a thin film is similar to a mesoscopic capacitor. A single-Cooper-pair tunnelling results in Josephson-vortex non-linear Bloch-type oscillations in a current-biased Josephson junction. We find the frequency and the amplitude of this motion.

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