Point-like vortices and magnetization relaxation in layered superconductors

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The energy of a point-like vortex is calculated for a layered superconductor with weak interlayer Josephson coupling. An energy barrier existing near the sample surface is found. The effect of the interlayer Josephson coupling on this barrier is considered. Magnetization relaxation due to thermally activated penetration of point-like vortices is considered. An initial avalanche-type decay of magnetization is predicted.

1. INTRODUCTION

The discovery of Bi and Tl based high-$T_c$ superconductors stimulated theoretical studies of layered superconductors with interlayer Josephson coupling. In particular, the specific point-like (or pancake) vortices were introduced and investigated [1-3]. An isolated point-like vortex cannot exist in the bulk of a macroscopic sample as its self-energy logarithmically diverges, when $L/\xi \rightarrow \infty$, where $L$ is the characteristic size of the sample, and $\xi$ is the coherence length.

Interaction of a point-like vortex with the sample surface consists of repulsion and attraction. The repulsion results from the interaction with the Meissner screening currents. The attraction results from the increase of the superconducting current density of the point-like vortex caused by the sample surface. The correlation between these two interactions is determined by the external magnetic field $H$. At a certain value of $H$, the competition of attraction and repulsion can lead to a stable state localized near the surface. The existence of this stable state affects the flux penetration process and, in particular, the magnetization relaxation.

In this paper, we study the penetration of point-like vortices into a layered superconductor. We show that the energy of a point-like vortex $G_v$ has a minimum $G_m$ detached by an energy barrier $G_g$ from the surface. We show that $G_m$ is negative if the external magnetic field $H$ is higher than a certain value $H_1$, which is of the order of the lower critical field $H_{c1}$. We estimate the effect of the interlayer Josephson coupling on the energy barrier $G_g$. We present a scenario of magnetization relaxation due to thermally activated penetration of point-like vortices inside the sample. We calculate the time dependence of the magnetization relaxation. We consider the case, when $\xi \ll \lambda$ and $d \ll \lambda$, where $\lambda$ is the London penetration depth, and $d$ is the distance between the superconducting layers.

2. SURFACE BARRIER

To calculate the magnetization relaxation, we find first the energy of a point-like vortex residing in one of the superconducting layers in the vicinity of the sample surface. We use for calculations the Lawrence-Doniach model [4], where we neglect the interlayer Josephson coupling.

Consider a semi-infinite layered superconductor subjected to a magnetic field $H$ parallel to the surface and perpendicular to the layers. Suppose the $z$-axis is parallel to $H$ and the $x$-axis is perpendicular to the surface. As we consider the case $\xi \ll \lambda$, the energy of a point-like vortex $G_v(x)$ is determined mostly by the magnetic field $B$ distribution into the sample.

For an isolated point-like vortex the magnetic field distribution $B(x)$ may be written as

$$B(x) = H \exp(-\frac{x}{\lambda}) + b(x)$$

(1)

Here the first term represents the magnetic field penetration into the sample in the absence of vortices. The magnetic field $b(x)$ results from the...
point-like vortex, and can be calculated by means of the method of images. It means that to the point-like vortex located at \((x, y)\) we add an image point-like antivortex located at \((-x, y)\) and take for \(b(r)\) the sum of the field due to the vortex and antivortex. Thus the field \(b(r)\) automatically vanishes on the sample surface and the boundary condition \(B(0, y, z) = H\) is satisfied. Using the magnetic field distribution, we calculate the isolated point-like vortex energy

\[
G_v = G_{va} + G_{vh}, \quad x > \xi, \quad \text{(2)}
\]

where

\[
G_{va} = \left(\frac{\Phi_0}{4\pi}\right)^2 d \ln \left(\frac{2xe}{\xi}\right), \quad x > \xi \quad \text{(3)}
\]

\[
G_{vh} = \frac{\Phi_0}{4\pi} H d \left[\exp\left(-\frac{x}{\lambda}\right) - 1\right], \quad x > \xi \quad \text{(4)}
\]

The term \(G_{va}\) represents the attraction between the point-like vortex and its image (point-like antivortex). It is minimal near the surface and increases monotonically with increase of \(x\). The term \(G_{vh}\) represents the repulsion of the point-like vortex from the surface due to the external magnetic field and the associated screening currents. It is maximal at \(x = 0\) and decreases monotonically with the increase of \(x\). The dependence of \(G_v\) on \(x\) increases monotonically if \(H < H_0\), where

\[
H_0 = \epsilon \frac{\Phi_0}{4\pi\lambda^2} \quad \text{(5)}
\]

For \(H > H_0\) the curve \(G_v(x)\) has a maximum \(G_s\) at \(x = x_s\) and a minimum \(G_m\) at \(x = x_m\). The explicit formulae for \(G_s, x_s, G_m,\) and \(x_m\) can be derived by means of Eqs (2)–(4) in the case, when \(H \gg H_0\) and \(\ln(\lambda/\xi) \gg 1\)

\[
G_s \approx \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 d \ln \left[\frac{\Phi_0}{2\pi} \frac{1}{\lambda \xi H}\right], \quad \text{(6)}
\]

\[
x_s \approx \frac{\Phi_0}{4\pi\lambda} \frac{1}{H} < \lambda, \quad \text{(7)}
\]

\[
G_m \approx \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 d \ln \left[2e\frac{\lambda}{\xi} \ln \left(\frac{4\pi\lambda^2}{\Phi_0} H\right)\right] - \frac{\Phi_0}{4\pi} dH \quad \text{(8)}
\]

\[
x_m \approx \lambda \ln \left(\frac{4\pi\lambda^2}{\Phi_0} H\right) > \lambda \quad \text{(9)}
\]

It follows from Eq (8) that the value of the minimum energy \(G_m\) becomes negative when \(H > H_1\), where

\[
H_1 \approx \frac{\Phi_0}{4\pi\lambda^2} \ln \left[2e\frac{\lambda}{\xi} \ln \left(\frac{\lambda}{\xi}\right)\right] \quad \text{(10)}
\]

Note, that the magnetic field \(H_1\) is higher than the lower critical field \(H_{c1}\).

Let us now estimate the effect of the interlayer Josephson coupling on the energy barrier \(G_s\) and the magnetic field \(H_1\). The interaction energy between the point-like vortex and antivortex \(G_{va}\) including the interlayer Josephson coupling was considered in [5]. It was shown that the value of \(G_{va}\) is determined by the interaction via magnetic field for \(r \ll \lambda_J\) and is determined by the interlayer Josephson coupling for \(r \gg \lambda_J\), here \(r\) is the distance between the point-like vortex and antivortex, and \(\lambda_J\) is the Josephson length. Applying these results to the interaction of the point-like vortex and its image we find that the value of \(G_{va}\) for \(x \ll \lambda_J\) is given by Eq (3) and for \(x \gg \lambda_J\) by the formula [5]

\[
G_{va} = 2\sqrt{2\pi} \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 d \frac{x}{\lambda_J}, \quad x \gg \lambda_J \quad \text{(11)}
\]

The space scale of \(G_{vh}\) is the penetration depth \(\lambda\). Thus, two characteristic limiting cases can be considered, \(r \in \lambda \ll \lambda_J\) and \(\lambda \gg \lambda_J\). The first one corresponds to the \(Tl\)-based high-\(T_c\) superconductors, and the second one corresponds to the \(Bt\)-based high-\(T_c\) superconductors [6]. The above results concerning \(H_0, H_1\), and \(G_s\) are valid if \(\lambda \ll \lambda_J\). Let us now consider the case, when \(\lambda \gg \lambda_J\). It follows then from Eqs (2)–(4) that the change of the field \(H_0\) is considerable and \(H_0\) is of the order of

\[
H_0 \sim \frac{\Phi_0}{4\pi\lambda\lambda_J} \quad \text{(12)}
\]

Using Eqs (2), (4) and (11) it can be shown that for \(H \gg H_0\) the value of \(x_s \ll \lambda_J\) and thus the energy barrier \(G_s\) is given by Eq (6). The change of the field \(H_1\) is also considerable, i.e.,

\[
H_1 = \frac{1}{2\sqrt{2}} \frac{\Phi_0}{\lambda\lambda_J} \ln \left(2\sqrt{2} \frac{\lambda\lambda_J}{\Phi_0} H\right) \quad \text{(13)}
\]
Thus, the interlayer Josephson coupling in case, when $\lambda \gg \lambda_f$ results mainly in increasing of the characteristic magnetic fields $H_0$ and $H_1$, and doesn’t change considerably the value of $G_g$.

3. MAGNETIZATION RELAXATION

We consider now the magnetization relaxation for magnetic field $H_1 < H < H^*$. In this case, the energy of an Abrikosov vortex is negative in the bulk. However, the Bean–Livingstone barrier for a nucleus of an Abrikosov vortex (vortex loop) is of the order of $\lambda G_g/d \gg G_g$ [8]. On the other hand, for the same magnetic field interval, the minimum energy of a point-like vortex $G_m$ is negative, and the energy barrier is equal to $G_g$. At non-zero temperature it can lead to a thermally activated penetration of the point-like vortices into a sample and thus to a specific mechanism of magnetization relaxation in layered superconductors. This mechanism is especially effective if the external magnetic field is low, i.e., when $H_1 < H < H^*$.

We consider here, as an illustration, the magnetization relaxation for the following problem: A semi-infinite layered superconductor is cooled down to a certain temperature $T$ below the critical temperature in zero magnetic field. Then, a magnetic field $H$ parallel to the sample surface is instantaneously turned on. We suppose that the superconducting layers are perpendicular to $H$ and the field is from the interval $H_1 < H < H^*$. We treat here the following scenario of magnetization relaxation:

The initial magnetization $M_0$ is equal to

$$M_0 = -\frac{1}{4\pi} H$$

(14)

The thermally activated penetration of point-like vortices into the sample lead to the decay of the magnetization $M$. The rate of this process is determined mostly by the energy barrier for the point-like vortices, and it depends exponentially on this barrier. After penetration into the sample the point-like vortices reside randomly in the superconducting layers in the vicinity of the plane $x = x_m$. The interaction of the incoming vortex with these vortices changes the energy barrier, thus, to find the magnetization relaxation rate we have to find this energy barrier shift $\delta G_g$.

The value of $\delta G_g$ is determined by the repulsion of the point-like vortices residing in the same layer and the attraction of the point-like vortices residing in different layers. To find the value of $\delta G_g$ we suppose that the point-like vortices inside the sample are distributed randomly along the line $x = x_m$ in each of the superconducting layers. The result of the calculation shows that $\delta G_g$ is proportional to the average linear concentration $N$ of the point-like vortices

$$\delta G_g = -\frac{Nd}{2\gamma_g M_0} \left(\frac{\Phi_0}{4\pi \lambda}\right)^3,$$

(15)

where $M_0 = H/4\pi$, and $\gamma_g$ is a number of the order of one. The main contribution to the decrease of the energy barrier results from the attractive interaction with the point-like vortices residing in the nearest $\lambda/d \gg 1$ layers. The entire number of these point-like vortices is of the order of $N x_m \lambda/d \gg 1$. The formula given by Eq (15) is valid if the average distance $l = N^{-1}$ between the point-like vortices in each of the superconducting layers is less than the penetration depth, i.e., $N \lambda < 1$.

Similar calculations show that the increase of the magnetization $\delta M$ due to the point-like vortices residing in the superconducting layers near the sample surface is also proportional to $N$

$$\delta M = N \frac{\Phi_0}{8\pi \lambda} \ln \left(\frac{4\pi \lambda^2}{\Phi_0} H\right)$$

(16)

Note, that Eq (16) is an exact one, i.e., it is valid for any values of $N$.

Finally, using the Eqs (15) and (16) we can present the dependence of $\delta G_g$ on $\delta M$ as

$$\delta G_g = -\frac{\delta M}{\gamma_g M_0} \left(\frac{\Phi_0}{4\pi \lambda}\right)^2 \ln^{-1} \left(\frac{4\pi \lambda^2}{\Phi_0} H\right)$$

(17)

Thus, the energy barrier decreases with the increase of the magnetization. This dependence results in the avalanche-type thermally activated magnetization relaxation.

To find the equation determining the magnetization relaxation we consider the diffusion of the
point-like vortices in the layer \( x < x_m \). To estimate the appropriate diffusion coefficient we treat the motion of the point-like vortices as a viscous flux flow. We consider also the dependence of the energy of the point-like vortices \( G_0 \) on \( x \) as an external potential. Finally, this approach results in the following equation

\[
\frac{dM}{dt} = \frac{\alpha M_0}{\tau} \exp \left( -\frac{\delta G_0}{k_B T} \right),
\]

where

\[
\alpha = \gamma \frac{k_B T}{\rho n c^2} \left( \frac{4\pi \lambda^2}{\Phi_0} \right)^2 \ln \left( \frac{4\pi \lambda^2}{\Phi_0} \right),
\]

\[
\tau = \frac{\lambda^2}{\rho n c^2} \exp \left( \frac{G_0}{k_B T} \right),
\]

\( \gamma \) is a number of the order of one, \( \rho n \) is the resistivity in the normal state, \( k_B \) is the Boltzman constant, and

\[
\tau_0 = \frac{\lambda^2}{\rho n c^2},
\]

is the characteristic time constant.

The solution of Eq (18) has the form

\[
M = M_0 \left[ -1 + \alpha \ln \left( \frac{\tau}{\tau - t} \right) \right]
\]

Thus, the thermally activated point-like vortices penetrate into the sample lead to a specific avalanche-type dependence of the magnetization on time. The characteristic time of the magnetization decay \( \tau \) strongly (exponentially) depends on the energy barrier and the temperature. The dimensionless amplitude \( \alpha \) of the magnetization relaxation is proportional to the temperature and slowly (logarithmically) depends on the applied magnetic field.

The dependence given by the Eq (22) is valid until

\[
\delta M = \alpha M_0 \ln \left( \frac{\tau}{\tau - t} \right) < M_0,
\]

and the density of vortices \( N \) is less than a certain critical value \( N_c \sim \lambda^{-1} \). When \( N \) becomes of the order of \( N_c \), Abrikosov vortices self-assemble from the point-like vortices and then penetrate inside the bulk. It follows from the Eqs (16) and (23), that a noticeable increase of \( N \) starts, when \( t \to \tau \). The penetration of Abrikosov vortices inside the bulk changes the time dependence of the magnetization decay to the regular logarithmic law. The latter leads to the magnetization relaxation rate decreasing in time. Thus, at \( t \approx \tau \) the magnetization relaxation rate has a maximum.

4. SUMMARY

We have shown that in an external magnetic field higher than \( H_0 \) the energy of a point-like vortex \( G_0 \) has a minimum \( G_m \) detached from the sample surface by an energy barrier \( G_s \). The value of \( G_m \) becomes negative in magnetic field higher than \( H_1 \). We have calculated the time dependence of the magnetization relaxation due to the thermally activated penetration of point-like vortices inside the sample. We have shown that this process results in an avalanche-type initial decay of the magnetization and a specific maximum in the magnetization relaxation rate at a certain moment of time \( t \approx \tau \). We have estimated that the interlayer Josephson coupling doesn’t change the obtained results considerably.

This research was supported in part by the Foundation Raschi.

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