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# Plastic strain and flux jumps in hard and composite superconductors

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**Abstract.** A study is made into the effect of the critical current density dependence upon the value of plastic strain on the critical state stability in hard and composite superconductors under conditions of plastic yield of the material. There have been found criteria of the critical state stability relative to the jointly developing magnetic flux jumps and plastic strain jerks.

## 1. Introduction

Magnetic flux jumps are known (Mints and Rakhmanov 1977) to be characteristic thermomagnetic instabilities of the critical state in hard superconductors and superconducting composites. Under conditions of plastic strain in superconductors, characteristic thermomagnetic instabilities in plastic yield of the material, i.e., plastic strain jerks, may occur simultaneously with magnetic flux jumps (Anashkin *et al* 1977, Schmidt and Pasztor 1978). The effect of plastic deformation on the critical state stability in superconductors has been studied theoretically by Mints (1980) and Maksimov and Mints (1981). As shown by these studies, the presence of plastic yield of the material may decrease considerably the threshold of critical state stability. Physically, this is due to the interaction of both instabilities (magnetic flux jumps and plastic strain jerks) stimulating each other by means of heat release. As a result of such processes, a new critical state instability occurs, namely, thermomagnetomechanical instability (Mints 1980).

It was assumed in the earlier papers by the present authors that the critical current density $j_c$ was independent of the value of strain $\varepsilon$, i.e., it was assumed that $\partial j_c/\partial\varepsilon=0$. This condition describes with good accuracy the situation observed in many superconducting materials (cf Koch and Easton 1977). In the present paper we derive a criterion of critical state stability with respect to thermomagnetomechanical disturbances (i.e., jointly developing magnetic flux jumps and plastic strain jerks) in the case of $(\partial j_c/\partial\varepsilon)\neq 0$, which is, for example, typical of superconductors of the A-15 type. We consider a plane–parallel plate having a thickness of $2b$ with a transport current of $I<I_c$ ($I_c=2bj_c$ is the critical current of the sample) under the various conditions of external heat removal.

## 2. Stability criterion

When accurately stated, the formulated problem amounts to a stability study of a system of Maxwell and heat equations describing the development of perturbations of the temperature $\theta$, electric field $E$ and strain $\delta\varepsilon$:

$$\left.\begin{aligned} \nabla^2 \boldsymbol{E} &= \frac{4\pi}{c^2}\frac{\partial j}{\partial t} \\ \nu\dot{\theta} &= \kappa\,\nabla^2\theta + \boldsymbol{jE} + \sigma\delta\dot{\varepsilon} \end{aligned}\right\} \tag{1}$$

where $\nu$ and $\kappa$ are the specific heat and thermal conductivity of the superconductor, respectively, and $\sigma$ is the mechanical stress. To a linear (in small perturbations) approximation, the current density $j$ can be represented as:

$$j = j_c(T_0) + \left(\frac{\partial j_c}{\partial T}\right)_\varepsilon \theta + \left(\frac{\partial j_c}{\partial \varepsilon}\right)_T \cdot \delta\varepsilon + \frac{1}{\rho}E \tag{2}$$

where $T_0$ is the coolant temperature, and $\rho$ is the superconductor resistivity in the flux-flow mode. Here and below, we shall assume for simplicity that the critical current

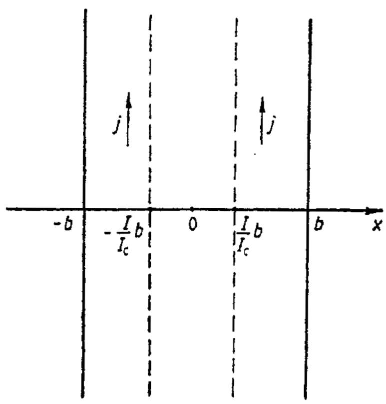


**Figure 1.** The sample geometry.

density does not depend on the magnetic field intensity $H$, i.e. $\partial j_c/\partial H = 0$. The critical state stability in the absence of plastic strain is then defined by the parameter

$$\beta = -\frac{4\pi}{c^2}\frac{j_c b^2}{\nu}\left(\frac{\partial j_c}{\partial T}\right)_\varepsilon$$

(Mints and Rakhmanov 1977).

In a plane problem (cf figure 1), the perturbations of interest to us are one-dimensional, $\theta = \theta(x)$, and the solutions to the system (1) can be conveniently found in the form:

$$\theta(x, t) = \theta_0\left(\frac{x}{b}\right)\exp\left(\lambda\frac{t}{t_\kappa}\right)$$

where $t_\kappa = \nu b^2/\kappa$ is the characteristic time of redistribution of heat in the sample, with the expressions for $E(x, t)$ and $\delta\varepsilon(x, t)$ having an analogous form. The critical state stability threshold depends evidently on the value of the parameter $\beta = \beta_c(\sigma)$ at which the increment of perturbation growth $\lambda$ satisfies the condition

$$\lambda = \lambda_c(\sigma) \geqslant 0. \tag{3}$$

Mints and Petukhov (1980) have found the relationship between $\delta\varepsilon$ and $\theta$ in the case of interest to us:

$$\delta\varepsilon=\left(\frac{\partial\dot{\varepsilon}}{\partial T}\right)_{\varepsilon}\frac{t_{\kappa}}{\lambda+\delta}\,\theta. \tag{4}$$

Here

$$\dot{\varepsilon}=\dot{\varepsilon}\,(T,\,\varepsilon) \tag{5}$$

is the rate of plastic yield of the material, and the parameter

$$\delta=t_{\kappa}\cdot\left|\left(\frac{\partial\dot{\varepsilon}}{\partial\varepsilon}\right)_{T}\right|$$

characterises the value of strain hardening in the course of plastic deformation. It is not difficult to estimate that for characteristic values of parameters (Startsev *et al* 1975), $\delta\sim10^{-4}\text{–}10^{-2}\ll1$.

Using the expression (4), one can represent the current density $j$ as

$$j=j_{\mathrm{c}}(T_0)+\left(\frac{\partial j_{\mathrm{c}}}{\partial T}\right)_{\varepsilon}(1+\tilde{\gamma})\;\theta+\frac{1}{\rho}\,E \tag{6}$$

where

$$\tilde{\gamma}=\gamma\left(1+\frac{\lambda}{\delta}\right)^{-1}\qquad\gamma=-\frac{(\partial j_{\mathrm{c}}/\partial\varepsilon)_{T}\cdot(\partial\dot{\varepsilon}/\partial T)_{\varepsilon}}{(\partial j_{\mathrm{c}}/\partial T)_{\varepsilon}\cdot(\partial\dot{\varepsilon}/\partial\varepsilon)_{T}}.$$

The parameter $\gamma$ defines the variation of critical current density with deformation of the material: as a rule, $\gamma>0$, because $(\partial j_{\mathrm{c}}/\partial\varepsilon)_{T}<0$, $(\partial\dot{\varepsilon}/\partial T)_{\varepsilon}>0$, $(\partial\dot{\varepsilon}/\partial\varepsilon)_{T}<0$. One may easily derive that

$$(1+\gamma)\left(\frac{\partial j_{\mathrm{c}}}{\partial T}\right)_{\varepsilon}=\left(\frac{\partial j_{\mathrm{c}}}{\partial T}\right)_{\dot{\varepsilon}}.$$

It is not difficult to evaluate the characteristic value of $\gamma$, for example, with the aid of a thermo-activation model of plastic yield, i.e. by selecting $\dot{\varepsilon}=\dot{\varepsilon}\,(T,\,\varepsilon)$ in the form

$$\dot{\varepsilon}=\dot{\varepsilon}_0\exp\left[-U(\varepsilon)/T\right].$$

Using the parameter values from the papers by Startsev *et al* (1975) and Koch and Easton (1977), we find that $\gamma\sim10^{-3}-10^{-2}\ll1$ (for Nb–Ti superconducting alloys) and $\gamma\lesssim1$ (for superconductors of the A-15 type).

It is seen from the current density expression (6) and system of equations (1) that the dependence of critical current density $j_{\mathrm{c}}$ on the value of strain $\varepsilon$ only affects the critical state stability to the extent of the value of $\tilde{\gamma}$ as compared to unity. The equations for the value of $\lambda_{\mathrm{c}}$ and the stability threshold for the various conditions of external heat removal can be derived from analogous equations found by Maksimov and Mints (1981) by substituting $\tilde{\beta}(\lambda_{\mathrm{c}})$ for $\beta$, where $\tilde{\beta}=\beta(1+\tilde{\gamma})$. Inasmuch as $\gamma\leqslant1$, the effect of interest to us may only turn out to be considerable when $\lambda_{\mathrm{c}}<\delta$ because in the opposite case ($\lambda_{\mathrm{c}}>\delta$) the inequality $\tilde{\gamma}\ll1$ is certain to be satisfied. Maksimov and Mints (1981) have studied the dynamics of perturbation development of temperature, electromagnetic field and plastic strain in the critical state in the presence of plastic yield of the material and have shown that the relation $\lambda_{\mathrm{c}}<\delta$ is only realised if the sample is thermally insulated. Therefore, the plastic strain dependence of the critical current density may only have a considerable effect upon the threshold of critical stability when adiabatic heat boundary conditions are realised on the sample surface. Note that in this case $\lambda_{\mathrm{c}}=0$ (Maksimov and Mints 1981).

Let us now proceed to find the criteria of critical state stability in hard ($\tau \ll 1$) and composite ($\tau \gg 1$) superconductors with respect to thermomagnetomechanical instability. Here

$$\tau = \frac{4\pi}{c^2} \frac{\kappa}{\nu \rho}.$$

Using the results obtained by Maksimov and Mints (1981), one can readily show that the critical state in hard superconductors ($\tau \ll 1$) is stable if

$$\left.\begin{array}{ll} \beta < \dfrac{\pi^2}{4} \left(\dfrac{I_c}{I}\right)^2 (1 + 2\sqrt{\tau}) & \sigma < \tilde{\sigma}_0 \\ \beta_0 < 3 \left(\dfrac{I_c}{I}\right)^3 \left(1 - \dfrac{\sigma}{\sigma_0}\right) & \sigma > \tilde{\sigma}_0 \end{array}\right\} \tag{7}$$

where

$$\tilde{\sigma}_0 = \sigma_0 \left(1 - \frac{\pi^2}{12} \frac{I}{I_c} (1 + 2\sqrt{\tau}) \frac{(\partial j_c/\partial T)_{\dot{\varepsilon}}}{(\partial j_c/\partial T)_{\varepsilon}}\right) \tag{8}$$

$$\sigma_0 = \frac{\nu}{(\partial \varepsilon/\partial T)_{\dot{\varepsilon}}} \tag{9}$$

and

$$\beta_0 = \beta (1 + \gamma) = -\frac{4\pi}{c^2} \frac{b^2 j_c}{\nu} \left(\frac{\partial j_c}{\partial T}\right)_{\dot{\varepsilon}}. \tag{10}$$

Analogously, in composite superconductors ($\tau \gg 1$) the critical state is stable if

$$\beta_0 < 3 \left(\frac{I_c}{I}\right)^3 \left(1 - \frac{\sigma}{\sigma_0}\right). \tag{11}$$

## 3. Discussion of the results

It is seen from the criteria (7) and (11) that the shift of stability threshold at $\gamma \sim 1$ may be quite large compared to the case $\gamma = 0$. This is especially true of the region of parameter values in which $\gamma < 0$ where, as seen from (7) and (11), the critical state stability increases. The condition $\gamma < 0$ is satisfied, in particular, if the critical current density grows with plastic strain, i.e., $(\partial j_c/\partial \varepsilon)_T > 0$. Experimentally, such a situation was observed over a range of plastic strain values in the paper by Rupp (1977), for example.

We shall consider in more detail another circumstance arising when $(\partial j_c/\partial \varepsilon)_T \neq 0$. In the absence of plastic yield of the material, the essential condition for the emergence of the critical state instability (magnetic flux jump) was the condition $(\partial j_c/\partial T)_\varepsilon < 0$ (Mints and Rakhmanov 1977). As seen from (7) and (11), the essential condition for the emergence of thermomagnetomechanical instability of the critical state in an adiabatically heat-insulated sample is the condition $(\partial j_c/\partial T)_{\dot{\varepsilon}} < 0$. Physically, the emergence of the derivative $(\partial j_c/\partial T)_{\dot{\varepsilon}}$ in the stability criterion is obvious because it is the value of $(\partial jc/\partial T)_{\dot{\varepsilon}}$ which determines the overall variation of the critical current density with slow temperature variations ($\lambda_c = 0$) in the course of plastic deformation. In the vicinity of, say, the peak-effect, the derivatives $(\partial j_c/\partial T)_\varepsilon$ and $(\partial j_c/\partial T)_{\dot{\varepsilon}}$ may differ from each considerably. Indeed, since

$$\left(\frac{\partial j_c}{\partial T}\right)_{\dot{\varepsilon}} = \left(\frac{\partial j_c}{\partial T}\right)_{\varepsilon} - \left(\frac{\partial j_c}{\partial \varepsilon}\right)_T \frac{(\partial \dot{\varepsilon}/\partial T)_\varepsilon}{(\partial \dot{\varepsilon}/\partial \varepsilon)_T} \tag{12}$$

it can be seen from equation (12) that for $(\partial j_c/\partial T)_\varepsilon > 0$ and $(\partial j_c/\partial T)_\varepsilon$ of not too high a value (in the vicinity of the peak-effect) a situation is possible where $(\partial j_c/\partial T)_{\dot{\varepsilon}} < 0$. Thus in this case, the critical state is stable with respect to magnetic flux jumps at $\sigma = 0$ ($(\partial j_c/\partial T)_\varepsilon > 0$) and unstable with respect to thermomagnetomechanical perturbations at some value of $\sigma$ selected from the $0 < \sigma < \sigma_0$ range ($(\partial j_c/\partial T)_{\dot{\varepsilon}} < 0$).

## 4. Conclusions

As shown in the present paper, the plastic strain dependence of the critical current density may lead to a considerable variation of the criterion of critical state stability with respect to thermomagnetomechanical perturbations for thermally insulated samples only. It has been found that, in this case, the condition $(\partial j_c/\partial T)_{\dot{\varepsilon}} < 0$ is essential for the emergence of critical state instability.

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