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On the theory of the training phenomenon in superconductors

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Abstract. The critical state stability is investigated taking into account the plastic yield of hard and composite superconductors. The conditions have been found at which the magnetic flux jumps occur, codeveloping with the plastic deformation jerks. It is shown that the interaction of mechanical and thermomagnetic instabilities can reduce appreciably the critical state stability threshold. The effect of the transport current on stability is investigated. The conditions of weak external cooling (in particular, the adiabatic case) are considered in this work. A possible explanation of the training phenomenon is suggested.

1. Introduction

The training and degradation phenomena in superconductors are well known both in magnetic systems and in short samples. As has been shown experimentally by Anashkin *et al* (1975), Schmidt (1976) and Schmidt and Pasztor (1977), the training phenomenon is strongly connected with the mechanical properties of the superconducting materials.

In the present paper we shall investigate the critical state stability taking into account the plastic yield of hard superconductors and superconducting composites. Plastic deformation jerks and flux jumps can occur in superconducting samples with the transport current under significant mechanical stress (resulting from either external or ponderomotive forces).

Plastic yield instabilities (the so-called serrated yielding) have been observed during mechanical loading of metals at low temperatures (see e.g. Basinski 1957, Klyavin 1974). The plastic deformation jerks are accompanied by strong heating, which gives rise to heat softening and results in strain hardening of the material. Plastic flow stability criteria have been obtained by Basinski (1957) for adiabatically insulated samples and by Kuramoto *et al* (1973) and Petukhov and Estrin (1975) for samples with intense external cooling. The plastic yield dynamics and the stability criterion have been considered by Mints and Petukhov (1980) for the case of arbitrary external cooling.

Flux jumps are inherent instabilities of the critical state. The nature of such thermomagnetic instabilities has been discussed earlier (see e.g. Mints and Rakhmanov 1977 and references therein) and the corresponding stability criteria are in good agreement with experiment.

However, the conditions under which the plastic deformation jerks develop simultaneously with the flux jumps have not yet been researched. Nevertheless, the interaction of both instabilities can considerably change the critical state stability criterion. The heat generation caused by the plastic deformation stimulates the flux jumps, which, in their turn, cause the plastic deformation jerks. Let us denote the duration time of the plastic deformation jerk as t_ϵ and the rise time of the flux jump as t_j. The interaction of the mechanical and thermomagnetic perturbations will, apparently, be most effective if $t_j \sim t_\epsilon$. The plastic yield instabilities develop slowly compared to the thermal diffusion time in the sample t_κ: $t_\epsilon \gg t_\kappa$ (Klyavin 1974, Mints and Petukhov 1980). In hard superconductors the slow flux jumps ($t_j > t_\kappa$) can occur only if the external cooling is weak (Maksimov and Mints 1980). In composites the strong interaction of thermomagnetic and mechanical instabilities takes place regardless of the intensity of cooling. Physically it is due to the slow development of both instabilities (t_j, $t_\epsilon \gg t_\kappa$). The instability criterion has been obtained by Mints (1980) for composites on the basis of the qualitative consideration.

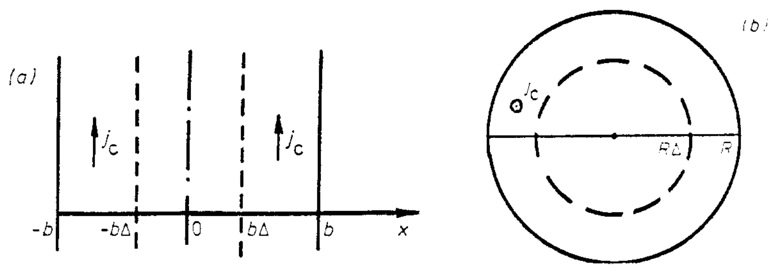

Figure 1. Sample geometry. (*a*) The flat plate; (*b*) the cylindrical wire with fixed transport current.

In this work we shall find the criterion of the mechanical and thermomagnetic (thermomagnetomechanical) instability under weak cooling of the external boundaries of the sample. The effect of the transport current I on the stability is investigated for a flat plate of finite thickness (figure 1 (*a*)) and for a cylindrical sample (figure 1 (*b*)). The theoretical results obtained in the present work are discussed in connection with the training phenomenon.

2. Basic equations

Let us denote the initial temperature of the superconductor as T_i and introduce θ, the small temperature disturbance ($\theta \ll T_c - T_i$ where T_c is the critical temperature of the superconductor). The sample temperature T_i depends on the external cooling conditions, mechanical stress σ and plastic deformation rate $\dot{\epsilon}$. In general, the instabilities we are interested in develop rapidly compared to the rate of change of the sample temperature $\dot{T}_i(\dot{T}_i t_\epsilon \ll T_c - T_i)$. So, the variation of the temperature T_i during the flux jump can be neglected. Note that the temperature T_i is almost uniform over the cross-section of the sample if the external cooling is weak.

The disturbances of the temperature θ, electric field $\boldsymbol{E}$ and deformation $\delta\epsilon$ are described by the heat diffusion equation and Maxwell equations:

$$\begin{cases} \nu\dot{\theta} = \kappa\, \nabla^2\theta + \boldsymbol{jE} + \sigma\delta\dot{\epsilon} \\ \text{curl curl } \boldsymbol{E} = -(4\pi/c^2)(\partial \boldsymbol{j}/\partial t) \end{cases} \tag{1}$$

where ν, κ are the heat capacity and the heat conductivity of the superconductor, and $\boldsymbol{j}$ is the current density. The last term in the first equation ($\sigma\delta\dot{\epsilon}$) describes the heat generation caused by the plastic deformation. In the case when only a part of the work of plastic deformation $\sigma\delta\dot{\epsilon}$ is converted into heat the necessary modification can be introduced to the final results by a correction factor. The heat generation jE_{i}, connected with the electric field E_{i} induced by the superconductor motion in the magnetic field, was omitted in equation (1). This term is small ($H_I^2/4\pi\sigma \ll 1$, H_I being the current self-magnetic field) compared to that directly describing mechanical heating $\sigma\delta\dot{\epsilon}$.

The critical current density depends only weakly on strain ϵ for a wide class of superconducting materials (Koch and Easton 1977); so we assumed here $\partial j_{\mathrm{c}}/\partial\epsilon = 0$. The expression for j assuming a linear depending on θ and E is the following:

$$j = j_{\mathrm{c}}(T_{\mathrm{i}}) + \rho_{\mathrm{f}}^{-1}E - \left|\frac{\partial j_{\mathrm{c}}}{\partial T}\right| \theta \tag{2}$$

where $j_{\mathrm{c}} = j_{\mathrm{c}}(T)$ is the critical current density, (for simplicity we are using Bean's (1964) critical state model: $\partial j_{\mathrm{c}}/\partial H = 0$), and ρ_{f} is the specific electrical resistivity of the superconductor in the flux flow regime.

Being interested in the critical state stability in the whole sample we shall regard the composite superconductor as a uniform anisotropic superconducting medium. The physical properties of such a medium are defined by superconducting filaments and normal conducting matrix characteristics averaged over the cross-section of the composite (see e.g. Mints and Rakhmanov 1977). Therefore, the simultaneous equations (1) and (2) are applicable in describing both the solid hard superconductors as well as superconducting composites.

Let the current j be parallel to the z axis. In this geometry (see figure 1) the perturbations depend only on the transverse† coordinate $\boldsymbol{r}$:

$$\boldsymbol{E} = [0, 0, E(\boldsymbol{r}, t)] \qquad \theta = \theta(\boldsymbol{r}, t) \qquad \delta\epsilon = \delta\epsilon(\boldsymbol{r}, t).$$

We assume the mechanical stress to be applied along the z axis.

We shall attempt the solution of the system of equations (1) in the following form:

$$\theta = \theta(\boldsymbol{r}) \exp[\lambda t(\kappa/\nu L^2)]$$

$$E = E(\boldsymbol{r}) \exp[\lambda t(\kappa/\nu L^2)]$$

$$\delta\epsilon = \delta\epsilon(\boldsymbol{r}) \exp[\lambda t(\kappa/\nu L^2)]$$

where λ is the eigenvalue to be defined and L is some typical length for each sample; for samples with an arbitrary shaped cross-section, $L \sim A/P$, where A and P are the area and the perimeter of the cross-section. As was shown by Mints and Petukhov (1980), the relation between $\delta\dot{\epsilon}$ and θ has the following form for the solutions we seek:

$$\delta\dot{\epsilon} = \frac{(\partial\dot{\epsilon}/\partial T)\,\theta}{1 + |\partial\dot{\epsilon}/\partial\epsilon|(\nu L^2/\kappa\lambda)} \tag{3}$$

where $\dot{\epsilon} = \dot{\epsilon}(T, \epsilon)$ is the plastic deformation rate.

Note that the plastic flow state may be stable only if $\partial\dot{\epsilon}/\partial\epsilon < 0$ (Mints and Petukhov 1980). It follows from equation (3), that the plastic heat generation $\sigma\delta\dot{\epsilon}$ does not depend on λ, provided $|\partial\dot{\epsilon}/\partial\epsilon|(\nu L^2/\kappa) = \delta \ll \lambda$. The parameter δ determines the strain hardening

† The situation with longitudinal nonuniformity (the perturbations depending on z) was considered by Mints (1980) on the basis of the qualitative theory.

magnitude of the deformable material. The typical value of δ for the samples with $L\sim 10^{-1}$–10^{-2} cm is $\delta\sim 10^{-2}$–$10^{-4}\ll 1$ (see e.g. Klyavin 1974).

With the system of equations (1) and the expressions for j and $\delta\dot{\epsilon}$, it is easy to obtain the equation for $\theta=\theta(\boldsymbol{x})$:

$$\nabla^2(\nabla^2\theta)-(\lambda\tau+\tilde{\lambda})\nabla^2\theta-\lambda(\beta-\tilde{\lambda}\tau)\theta=0. \tag{4}$$

Here

$$\tilde{\lambda}=\lambda\left(1-\frac{\alpha}{\lambda+\delta}\right) \qquad \alpha=\sigma\frac{\partial\dot{\epsilon}}{\partial T}\frac{L^2}{\kappa}$$

$$\beta=\frac{4\pi}{c^2}j_c\left|\frac{\partial j_c}{\partial T}\right|\frac{L^2}{\nu} \qquad \tau=\frac{4\pi}{c^2}\frac{\kappa}{\rho_t\nu}=\frac{D_t}{D_m}$$

where D_t and D_m are thermal and magnetic diffusivities respectively. The differentiation is carried out with respect to the dimensionless variable $\boldsymbol{x}=\boldsymbol{r}/L$. The parameter α determines the plastic flow stability with regard to the deformation jerks in the absence of the critical state (Petukhov and Estrin 1975). Correspondingly, the parameter β determines the critical state stability with respect to the flux jumps without the plastic yield (Mints and Rakhmanov 1977). The values of τ are $\tau\ll 1$ for hard superconductors and $\tau\gg 1$ for composites.

The relation between the electric field E and temperature θ can be easily found from the expressions (1), (2) and (3):

$$E=\begin{cases}0 & 0\leqslant|\boldsymbol{x}|\leqslant\Delta\\ (\kappa/j_cL^2)(\tilde{\lambda}\theta-\nabla^2\theta) & \Delta\leqslant|\boldsymbol{x}|\leqslant 1.\end{cases} \tag{5}$$

The region $\Delta\leqslant|\boldsymbol{x}|\leqslant 1$ corresponds to the current-carrying layer in the superconductor. For a flat sample $\Delta=\Delta(i)=1-i$, where $i=I/I_c$; I_c is the critical current (figure 1(*a*)). For a cylindrical wire $\Delta(i)=(1-i)^{1/2}$ (figure 1(*b*)). The equation for θ in the region $|\boldsymbol{x}|<\Delta$ has the form:

$$\nabla^2\theta=\tilde{\lambda}\theta. \tag{6}$$

To determine $\theta(\boldsymbol{x})$ and the eigenvalue spectrum $\lambda=\lambda(\beta,\alpha\ldots)$ equation (4) should be supplemented with four boundary conditions. The heat boundary condition on the sample surface is

$$\nabla\theta+W\theta|_{|\boldsymbol{x}|=1}=0 \tag{7}$$

where $W=W_0L/\kappa$, W_0 being the coefficient of heat transfer from the superconductor to the coolant. As for the electrodynamic condition on the surface, we assume that the external magnetic field remains constant during the process, i.e.

$$-\frac{1}{c}\frac{\partial\boldsymbol{H}}{\partial t}\bigg|_{|\boldsymbol{x}|=1}=\operatorname{curl}\boldsymbol{E}\bigg|_{|\boldsymbol{x}|=1}=\frac{\kappa}{j_cL^2}\operatorname{curl}[\boldsymbol{e}(\tilde{\lambda}\theta-\nabla^2\theta)]|_{|\boldsymbol{x}|=1}=0 \tag{8}$$

where $\boldsymbol{e}$ is the unit vector in the electric field direction.

The temperature and the heat flux are continuous at $|\boldsymbol{x}|=\Delta$. Then, using the solution of equation (6) one can obtain the following condition at $|\boldsymbol{x}|=\Delta$:

$$\nabla\theta-\tilde{W}\theta|_{|\boldsymbol{x}|=\Delta}=0 \tag{9}$$

where

$$\tilde{W}=(\nabla\theta/\theta)|_{|\boldsymbol{x}|=\Delta-0}$$

The heat impedance $\tilde{W}$ characterises the intensity of the heat transfer inside the superconductor. The electric field is also continuous at $|\boldsymbol{x}| = \Delta$. Hence, we get

$$\tilde{\lambda}\theta - \nabla^2\theta|_{|x|=\Delta} = 0. \tag{10}$$

The requirement of the existence of a nontrivial solution of equation (4) with the corresponding boundary conditions (7)–(10) allows us to determine the function $\lambda = \lambda(\beta, \alpha \ldots)$. The eigenvalue spectrum depends on the geometrical parameters L, $\Delta(i)$, the parameters of the material τ, δ and the external cooling. (If the mechanical stress is caused by the ponderomotive force $(1/c)\,[\boldsymbol{jH}] \neq 0$, then $\sigma = \sigma(i)$ and consequently $\alpha = \alpha(i)$.) The thermomagnetomechanical instability criterion is determined by the values α and β at which the positive (or equal to zero) eigenvalue appears first

$$\lambda(\beta, \alpha) = \lambda_c(\tau, i, \delta, W). \tag{11}$$

The condition (11) defines a certain dependence $\beta = \beta_c(\alpha)$ (or $\alpha = \alpha_c(\beta)$) determining the critical state (or the plastic yield) stability threshold. Evidently, the system is stable if $\beta < \beta_c(\alpha)$ (or $\alpha < \alpha_c(\beta)$).

3. Adiabatic boundary conditions

One may omit the temperature dependence of the coefficients in equation (4), if the temperature is uniform over the cross-section of the sample. Hence equation (4) becomes a fourth-order differential equation with constant coefficients. The general equation, obtained from equation (4) together with the conditions of equations (7)–(10), allows us to determine the eigenvalue spectrum $\lambda = \lambda(\beta, \alpha)$. In this section the dynamics and the stability criteria for the flat and cylindrical samples are considered under the condition of adiabatic thermal insulation.

3.1. The flat plate

For the sample with the geometry shown in figure 1(*a*) equation (4) has the following form:

$$\theta^{\text{iv}} - (\lambda\tau + \tilde{\lambda})\theta^{\text{ii}} - \lambda(\beta - \tilde{\lambda}\tau)\theta = 0. \tag{12}$$

The solutions we seek are, obviously, symmetrical relative to the x axis: $\theta(x) = \theta(-x)$. Therefore it is sufficient here to write down the boundary conditions at $x = 1$ and $x = \Delta = 1 - i$. On the sample surface ($x = 1$) they are as follows:

$$\theta^{\text{i}}(1) = 0 \qquad \tilde{\lambda}\theta^{\text{i}}(1) - \theta^{\text{iii}}(1) = 0.$$

By using the solution of equation (6)

$$\theta = C \cosh(\tilde{\lambda}^{1/2}x)$$

it is easy to obtain the heat impedance $\tilde{W}$ at $x = 1 - i$:

$$\tilde{W} = (\theta^{\text{i}}/\theta)|_{x=1-i-0} = \tilde{\lambda}^{1/2} \tanh[\tilde{\lambda}^{1/2}(1 - i)].$$

Hence the boundary conditions at $x = 1 - i$ are

$$\theta^{\text{i}} - \tilde{W}\theta|_{x=1-i} = 0 \qquad \tilde{\lambda}\theta - \theta^{\text{ii}}|_{x=1-i} = 0.$$

Equation (12) combined with the required boundary conditions has a nontrivial solution, if

$$k_1(k_2^2+\tilde{\lambda})\tanh(k_1 i)-k_2(k_1^2-\tilde{\lambda})\tan(k_2 i)+\tilde{W}(k_1^2+k_2^2)=0 \qquad (13)$$

where

$$k_{1,2}^2=\pm\frac{(\tilde{\lambda}+\lambda\tau)}{2}+\left(\lambda(\beta-\tilde{\lambda}\tau)+\frac{(\tilde{\lambda}+\lambda\tau)^2}{4}\right)^{1/2}.$$

In the general case equation (13) can only be solved numerically. Nevertheless, equation (13) allows the analytical solution in some limits. We shall analyse the eigenvalue spectrum $\lambda=\lambda(\beta,\alpha,\tau,\delta)$ in hard ($\tau\ll 1$) and composite ($\tau\gg 1$) superconductors.

Let us consider first the hard superconductors ($\tau\ll 1$). The numerical solution of equation (13) allows us to describe the evolution of the function $\lambda=\lambda(\beta)$ with the increase of α (see figure 2(*a*), curves A–F). The dependence $\lambda=\lambda(\beta)$ undergoes the most noticeable change in the region of slow ($\lambda\to 0$) perturbations. However, up to $\alpha=\alpha_2$ (the curve

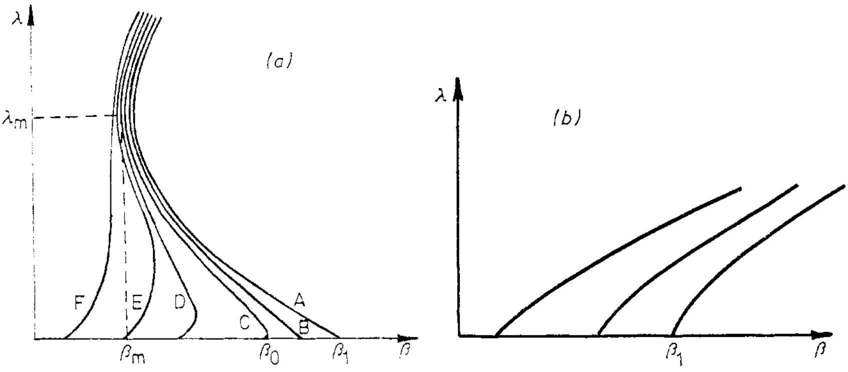

Figure 2. The function $\lambda=\lambda(\beta)$ for adiabatic thermal conditions at different α. (*a*) $\tau<\tau_0$: A, $\alpha=0$; B, $\alpha<\alpha_1$; C, $\alpha=\alpha_1$; D, $\alpha_1<\alpha<\alpha_2$; E, $\alpha=\alpha_2$; F, $\alpha_2<\alpha$. (*b*) $\tau>\tau_0$.

E on figure 2(*a*)) $\beta_0>\beta_m$ and the stability is violated by the fast ($\lambda i^2\gg 1$) perturbations†. Within this range of parameters the function $\beta=\beta(\lambda)$ has the following form:

$$\beta=\frac{\pi^2}{4i^2}-\alpha+\lambda\tau+\left(\frac{\pi^2}{4i^2}-\alpha\right)\cdot\frac{\pi^2}{4}\cdot\frac{1}{\lambda}. \qquad (14)$$

From equation (14) it follows:

$$\left.\begin{aligned}\lambda_c=\lambda_m&=\frac{\pi^2}{4i^2}\left(\frac{1-(4\alpha i^2/\pi^2)}{\tau}\right)^{1/2}\\ \beta_c=\beta_m(\alpha)&=\frac{\pi^2}{4i^2}(1+2\tau^{1/2})-\alpha.\end{aligned}\right\} \qquad (15)$$

In the case under consideration ($W=0$) the parameter α changes within the range $0\leqslant\alpha\leqslant\delta\leqslant 1$. Hence, from the expressions (14) and (15) we come to the conclusion that

† If $i<1$, then the thermal diffusion time in the current-carrying layer $t_\kappa(i)$ is of the order $t_\kappa(i)\simeq(\nu L^2/\kappa)i^2$ (Mints and Rakhmanov 1977). The ratio $t_\lambda/t_\kappa(i)\sim 1/(\lambda i^2)$ represents the measure of the rate of the process, where $t_\lambda=\nu L^2/\kappa\lambda$ is the rise time of the disturbance. Hereafter we shall define the fast perturbations by the condition $t_\lambda<t_\kappa(i)$ (or $\lambda i^2>1$) and the slow ones by the condition $t_\lambda>t_\kappa(i)$ (or $\lambda i^2<1$).

the eigenvalue spectrum is affected insignificantly in the fast perturbation region (because $\alpha i^2 \ll 1$). The presence of the normal current in the superconductor (described by the term $\lambda\tau$ in equation (14)) is the main stabilising factor with respect to the disturbances with $\lambda \to \infty$ (Mints and Rakhmanov 1975). Therefore, the plastic yield accounts only for small corrections in the stability criterion with respect to the fast perturbations ($\lambda i^2 \gg 1$).

On the contrary, the development of the slow ($\lambda i^2 \ll 1$) disturbances essentially depends on α. The point $\beta(\lambda=0)=\beta_0$ considerably displaces on the β, λ plane (see figure 2(a)) with the increase of α from $\alpha=0$ to $\alpha=\delta$. Let us determine the parameters α_1, α_2 and β_2. Expanding equation (13) in the power series of λi^2 ($\lambda i^2 \ll 1$), it is easy to show that λ becomes zero at

$$\beta=\beta_0=(3/i^3)[1-(\alpha/\delta)].$$

The value $\alpha=\alpha_2$ is defined by the condition $\beta(\alpha)=\beta_{\mathrm{m}}(\alpha)$ (figure 2(a), curve E), from which one finds

$$\alpha=\alpha_2=\delta\,\frac{1-i(\pi^2/12)(1+2\tau^{1/2})}{1-(i^3\delta/3)}. \tag{16}$$

The investigation of equation (13) in the vicinity of $\beta=\beta_0$ enables us to obtain the dependence $\beta=\beta(\lambda)$:

$$\beta/\beta_0=1+0{\cdot}4\lambda i^2(\tau-\tau_{\mathrm{c}}) \tag{17}$$

where

$$\begin{aligned}\tau_{\mathrm{c}}&=\tau_{\mathrm{c}}(i,\delta,\alpha)\\&=\left(1-\frac{\alpha}{\delta}\right)\tau_0-\frac{5}{2}\frac{\alpha}{\delta(\delta-\alpha)}\frac{1}{i^2}\end{aligned} \tag{18}$$

and $\tau_0=\tau_{\mathrm{c}}\,(\alpha=0)$ is (Maksimov and Mints 1980):

$$\tau_0(i)=(5/6i^2)-(9/7i)+\tfrac{1}{2}.$$

The parameter $\alpha=\alpha_1$ (see curve C, figure 2(a)) is defined by the condition

$$(\mathrm{d}\beta/\mathrm{d}\lambda)|_{\lambda=0}=0,$$

which is equivalent to the equality $\tau=\tau_{\mathrm{c}}\,(\alpha_1)$. From the expression (18) one finds:

$$\alpha_1=\tfrac{2}{5}\,(\tau_0-\tau)\,\delta^2 i^2.$$

The comparison of α_1 and α_2 shows us that $(\alpha_1/\alpha_2)\sim\delta i^2(\tau_0-\tau)\ll 1$ for all values of i. Thus it follows from equation (17), $(\mathrm{d}\beta/\mathrm{d}\lambda)|_{\lambda\to 0}>0$ if $\alpha\sim\alpha_2\gg\alpha_1$. The stability criterion with respect to small thermomagnetomechanical perturbations has the following form at $\alpha>\alpha_2$ (see curve F, figure 2(a)):

$$(\beta/\beta_1)+(\alpha/\delta)<1 \tag{19}$$

where

$$\beta_1(i)=3/i^3. \tag{20}$$

Condition (19) can be rewritten in the convenient form:

$$(i^3\beta/3)+(\sigma/\sigma_0)<1$$

by substituting the parameters α, δ and β_0; here

$$\sigma_0=\nu\left|\frac{\partial\dot{\epsilon}}{\partial\epsilon}\right|\Big/\left(\frac{\partial\dot{\epsilon}}{\partial T}\right).$$

Now it is easy to find the dependence $\beta=\beta_c(\alpha)$ by combining equations (15) and (19):

$$\begin{cases}\beta_2-\alpha-\beta=0 & \alpha<\alpha_2\\ 1-(\alpha/\delta)-(\beta/\beta_1)=0 & \alpha>\alpha_2\end{cases}\tag{21}$$

where

$$\beta_2=\beta_m(\alpha=0)=(\pi^2/4i^2)(1+2\tau^{1/2}).\tag{22}$$

For $\tau<\tau_0(i)$ the function $\beta=\beta_c(\alpha)$ is shown on the figure 3(*a*) with the parameters α_2, β_1 and β_2, defined by the expressions (16), (20) and (22), respectively. The presence of the plastic yield essentially diminishes the critical state stability threshold, provided the external stress is sufficiently large.

Let us consider now the composite superconductors ($\tau\gg 1$). The evolution of the function $\lambda=\lambda(\beta)$ with the increase of α is shown in figure 2(*a*) (for $\tau<\tau_0$) and in figure 2(*b*) (for $\tau>\tau_0$). For $\tau<\tau_0$ the behaviour of the curves $\lambda=\lambda(\beta,\alpha)$ does not differ qualitatively from the analogous one for hard superconductors (figure 2(*a*), curves A–F). This is due to the fact that the heat transfer inside the sample is the main stabilising factor with regard to the slow flux jumps in the absence of the external cooling. As shown by Maksimov and Mints (1980)

$$\lambda_m=2\cdot 5/(i^2\tau)$$

$$\beta_2=\beta_m(\alpha=0)=3\cdot 8\,\tau^{1/2}/i^2$$

if $i^2\tau\ll 1$, $\alpha=0$. It is easy to obtain β_m from equation (13) in the parameter range $\alpha\neq 0$, $\tau\ll\tau_0$:

$$\beta_m(\alpha)=\beta_2[1-(\alpha/2\lambda_m)].\tag{23}$$

As can be seen from equation (23), the correction to $\beta_m(\alpha=0)$ is small (being of the order $\alpha/\lambda_m\sim\alpha i^2\tau\ll 1$). This is connected with the weak interaction of the mechanical and thermomagnetic perturbations. The flux jump rise time $t_j=t_\kappa/\lambda_m\ll t_\kappa$ is much smaller than t_ϵ (see Introduction).

Let us determine the parameter α_2 for this case. The dependence $\beta=\beta(\lambda)$ is defined by the expression (17) provided $\lambda i^2\ll 1$. The value of the parameter $\beta_0=\beta_1[1-(\alpha/\delta)]$ depends significantly on α. The parameter α_2 is defined by the condition $\beta_0(\alpha_2)=\beta_m(\alpha_2)$ (figure 2(*a*), curve E) from which it is easy to obtain:

$$\alpha_2=\delta\,\frac{1-(i^3\beta_2/3)}{1-(i^3\delta\tau^{3/2}/4)}.$$

If $\alpha>\alpha_2$ (curve F, figure 2(*a*)), then $\beta_0(\alpha)<\beta_m(\alpha)$ (and correspondingly $\lambda_c=0$). Thus the stability criterion for $\alpha>\alpha_2$ is determined by the inequality (19) in the case $\tau<\tau_0$; so one can find the dependence $\beta=\beta_c(\alpha)$ combining the expressions (19) and (23) in the following form:

$$\begin{cases}(\beta/\beta_2)+(\alpha/2\lambda_m)=1 & \alpha<\alpha_2\\ (\beta/\beta_1)+(\alpha/\delta)=1 & \alpha>\alpha_2.\end{cases}\tag{24}$$

The function $\beta=\beta_c(\alpha)$ is shown in figure 3(*a*) with the corresponding parameters β_1, β_2 and α_2.

For $\tau>\tau_0$ from expression (18) it follows that $\tau-\tau_c>0$, and consequently $\lambda_c=0$ (see figure 2(*b*)), for any α. Hence, the criterion of thermomagnetomechanical instability is

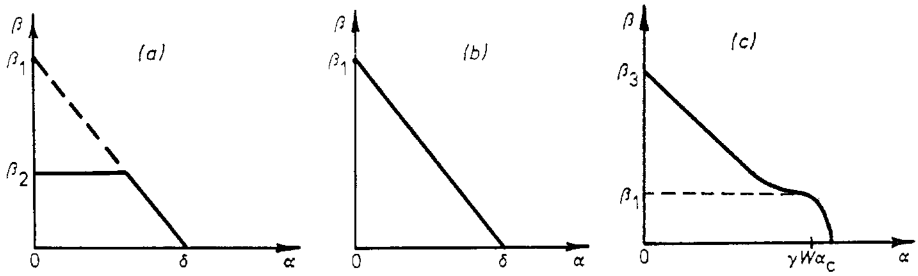

Figure 3. The function $\beta=\beta_c(\alpha)$. (*a*) $\tau<\tau_0$, $W=0$; (*b*) $\tau>\tau_0$, $W=0$; (*c*) $\tau\gg\tau_0$, $W\neq 0$ ($Wi^2\tau>1$).

determined by the condition

$$(\beta/\beta_1)+(\alpha/\delta)=1. \tag{25}$$

Figure 3(*b*) shows the function β_c versus α. The quench current (stress) depends strongly on the external stress (transport current) value. Such a significant correlation was experimentally observed by Pasztor and Schmidt (1977).

The evolution of the dependence $\beta=\beta_c(\alpha)$ with the decrease of the transport current is shown in figure 4 (curves A–D). It is seen that β_c is a linear function of α, provided $i>i_0$ where i_0 is defined by the equality $\tau_0(i_0)=\tau$. It can be shown, that figure 4 describes the evolution of the curves $\beta=\beta_c(\alpha)$ for superconductors with the parameter τ satisfying the inequality $\tau>\tau_0\,(i=1)=\frac{1}{21}$.

Note, that the criterion (25) does not depend on $\dot{\epsilon}$ for the thermoactivation model of the plastic yield:

$$\dot{\epsilon}=\dot{\epsilon}_0\exp\{-[u(\epsilon)/T]\}$$

if $\dot{\epsilon}_0$ does not depend on ϵ.

3.2. *The cylindrical sample*

For the samples with the configuration shown in figure 1(*b*), the perturbations do not depend on ϕ and the fourth-order equation (4) can be reduced to two independent

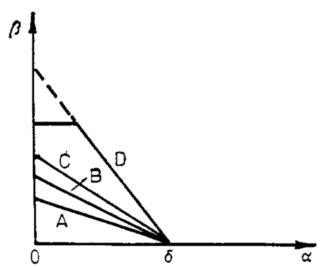

Figure 4. The evolution of the dependence $\beta=\beta_c(\alpha)$ with decrease of the transport current: A, $i=1$, B, $1>i>i_0$; C, $i=i_0$; D, $i_0>i$; for a flat plate.

second-order differential equations (Mints and Rakhmanov 1975) with respect to $|x|=r$:

$$\left.\begin{aligned}&\frac{d^2F_1}{dr^2}+\frac{1}{r}\frac{dF_1}{dr}-k_1^2F_1=0\\&\frac{d^2F_2}{dr^2}+\frac{1}{r}\frac{dF_2}{dr}+k_2^2F_2=0\\&\theta=F_1+F_2.\end{aligned}\right\} \tag{26}$$

The solution of each equation (26) is the zero-order Bessel function. The general solution for θ can be written in the form

$$\theta=c_1J_0(k_2r)+c_2N_0(k_2r)+c_3I_0(k_1r)+c_4K_0(k_1r)$$

where J_0 and N_0 are Bessel functions of the first and second kind, and I_0 and K_0 are the modified Bessel functions of the first and second kind. For a wire with a fixed transport current we set $L=R$ (figure 1(*b*); then $I=\pi R^2(1-\Delta^2)j_c$, $I_c=\pi R^2 j_c$ and $\Delta(i)=(1-i)^{1/2}$. The required boundary conditions have the form:

$$\frac{d\theta}{dr}=0 \qquad \frac{d}{dr}\left[\tilde{\lambda}\theta-\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)\right]=0 \qquad r=1$$

$$\frac{d\theta}{dr}-\tilde{W}\theta=0 \qquad \tilde{\lambda}\theta-\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)=0 \qquad r=\Delta(i)$$

where

$$\tilde{W}=\tilde{\lambda}^{1/2}\frac{I_1(\tilde{\lambda}^{1/2}\Delta)}{I_0(\tilde{\lambda}^{1/2}\Delta)}$$

Here I_1 is the modified first-order Bessel function of the first kind.

The nontrivial solution of the system of equations (26) with the corresponding boundary conditions exists, if

$$\begin{aligned}&k_1(k_2^2+\tilde{\lambda})\frac{I_1(k_1)K_1(k_1\Delta)-I_1(k_1\Delta)K_1(k_1)}{I_0(k_1\Delta)K_1(k_1)+I_1(k_1)K_0(k_1\Delta)}\\&\qquad-k_2(k_1^2-\tilde{\lambda})\frac{J_1(k_2)N_1(k_2\Delta)-J_1(k_2\Delta)N_1(k_2)}{J_0(k_2\Delta)N_1(k_2)-J_1(k_2)\,N_0(k_2\Delta)}+\tilde{W}(k_1^2+k_2^2)=0.\end{aligned} \tag{27}$$

Let us consider the case $\tau\ll 1$ (the hard superconductors). The evolution of the eigenvalue spectrum $\lambda=\lambda(\beta,\alpha)$ for cylindrical samples is qualitatively the same as in the flat geometry (see figure 2(*a*)). The system is stable against fast growing disturbances ($\lambda i^2=1$), if

$$\beta<\beta_m(\alpha)=k_m^2(1+2\tau^{1/2})-\alpha \tag{28}$$

where $k_m=k_m(i)$ is the first positive root of the equation:

$$J_0[x(1-i)^{1/2}]N_1(x)-J_1(x)N_0[x(1-i)^{1/2}]=0.$$

As was mentioned above, the noticeable change of the function $\lambda=\lambda(\beta,\alpha)$ occurs at $\alpha\sim\delta\ll 1$. Therefore, criterion (28) is practically unchanged with increase in α. The most considerable modification of the eigenvalue λ occurs in the slow perturbation region ($\lambda i^2\ll 1$). In this range of the parameters equation (27) permits an analytical solution, which allows us to investigate the behaviour $\beta(\lambda)$ at $\lambda\to 0$. Assuming $k_1 i$, $k_2 i\ll 1$ it is possible to represent the ratio

$$\frac{J_1(k_2)N_1(k_2\Delta)-J_1(k_2\Delta)N_1(k_2)}{J_0(k_2\Delta)N_1(k_2)-J_1(k_2)N_0(k_2\Delta)} \simeq \frac{k_2}{2(1-i)^{1/2}}\left[i+A(i)k_2^2+B(i)k_2^4+C(i)k_2^6\right] \tag{29}$$

where

$$A(i)=-\tfrac{1}{4}\ln(1-i)-[i(2+i)]/8$$

$$B(i)=\tfrac{1}{16}\ln^2(1-i)+[(1+2i)/16]\ln(1-i)+[i(6+9i+2i^2)]/96$$

$$C(i)=-\frac{1}{64}\ln^3(1-i)-\frac{(5+4i)}{128}\ln^2(1-i)-\frac{5(1+12i+6i^2)}{6\times 128}\ln(1-i)-\frac{i(60+390i+236i^2+33i^3)}{72\times 128}.$$

Similarly, for the modified Bessel functions combination:

$$\frac{I_1(k_1)K_1(k_1\Delta)-I_1(k_1\Delta)K_1(k_1)}{I_0(k_1\Delta)K_1(k_1)+I_1(k_1)K_0(k_1\Delta)} \simeq \frac{k_1}{2(1-i)^{1/2}}\left[i-A(i)k_1^2+B(i)k_1^4-C(i)k_1^6\right]. \tag{30}$$

In the expansions (29) and (30) we retain four terms to ensure the required accuracy.

Let us now determine the parameters α_1, α_2, β_0 and β_2. Substituting equations (29) and (30) into equation (27) one can find that $\lambda(\beta_0)=0$, where β_0 is defined by the expression

$$\beta_0=\beta_1\left[1-(\alpha/\delta)\right]$$

with the parameter β_1 given by

$$\beta_1=1/A(i). \tag{31}$$

The value $\alpha=\alpha_2$, at which $\beta_0(\alpha)=\beta_m(\alpha)$ (curve E, figure 2(*a*)) is

$$\alpha=\alpha_2=\delta\,\frac{1-\beta_2 A(i)}{1-\delta A(i)}$$

where

$$\beta_2=\beta_m(\alpha=0)=k_m^2(1+2\tau^{1/2}).$$

In the vicinity of $\beta=\beta_0$ the dependence $\beta=\beta(\lambda)$ is the following (with the accuracy $\lambda i^2 \ll 1$):

$$\beta/\beta_0=1+\lambda(B/A)(\tau-\tau_c) \tag{32}$$

where

$$\left.\begin{aligned}\tau_c&=\left(1-\frac{\alpha}{\delta}\right)\tau_0-\frac{\alpha}{\delta(\delta-\alpha)}\frac{A}{B}\\ \tau_0&=\frac{1}{BA}\left[A^3-2AB+C+\frac{(1-i)^2A}{8}\right].\end{aligned}\right\} \tag{33}$$

If the transport current is small ($i \ll 1$), the specific features of the cylindrical geometry are insignificant. By expanding the coefficients $A(i)$, $B(i)$ and $C(i)$ in the power series of i, it is easy to find that equations (32) and (33) coincide with the expressions (17) and (18) obtained for a flat sample.

The parameter α_1 can be found from the condition $(\mathrm{d}\beta/\mathrm{d}\lambda)|_{\lambda\to 0}=0$ (figure 2(*a*), curve C) by means of expressions (32) and (33):

$$\alpha_1=(B/A)(\tau_0-\tau)\delta^2.$$

As stated above, the coefficient $\tau-\tau_c$ in equation (32) is positive at $\alpha\gtrsim\alpha_2\gg\alpha_1$ (figure 2(*a*), curve F). Hence, the stability region is determined by the inequality (19) if $\alpha>\alpha_2$. The stability threshold $\beta=\beta_c(\alpha)$ is determined by equation (21), for hard superconductors ($\tau\ll 1$), with the parameters α_2, β_1 and β_2, corresponding to the cylindrical geometry (figure 3(*a*)).

The evolution of the eigenvalue spectrum $\lambda=\lambda(\beta, \alpha \ldots)$ in superconducting composites ($\tau\gg 1$) is shown in figure 2(*a*) (for $\tau<\tau_0$) and in figure 2(*b*) (for $\tau>\tau_0$).

In the case $1\ll\tau<\tau_0$ the peculiarities of the cylindrical geometry are not essential, since the transport current is small, i.e. $i\leqslant\tau^{-1/2}\ll 1$. Therefore, the results obtained in the preceding §3.1 for a flat sample are valid here.

Figure 2(*b*) shows the function $\lambda=\lambda(\beta)$ at different α for $\tau>\tau_0$. The qualitative behaviour of the curves $\lambda=\lambda(\beta)$ does not change with the increase of α. The instability criterion is determined by equation (25), since $\tau>\tau_c$ (and accordingly $\lambda_c=0$) for all α. Figure 3(*b*) shows the dependence $\beta=\beta_c(\alpha)$.

The evolution of the function $\beta=\beta_c(\alpha, i)$ with decrease of the transport current is qualitatively the same as in the flat geometry (see figure 4).

4. Weak external cooling

It was mentioned above that the temperature is almost uniform over the cross-section of the sample, if the external cooling is weak ($W\ll 1$). Employing this fact we can obtain the equation to determine the eigenvalue spectrum $\lambda=\lambda(\beta, \alpha \ldots)$ by means of the procedure used in §3. In the general case, this equation can only be solved numerically. Nevertheless, an analytical solution is possible in the most interesting range of the parameters: $\lambda i^2\ll 1$, $W\ll 1$. In this section we shall consider the quasiadiabatic case ($W\ll\delta$) and the conditions of dynamic stabilisation ($Wi^2\tau\gg 1$) for superconducting composites.

4.1. Quasiadiabatic stability criterion

It is shown by Mints and Rakhmanov (1977), that $\beta(\lambda=0)=\infty$ (and $\lambda_c\neq 0$), provided $W\neq 0$. The presence of weak cooling can considerably change the dynamics of the instabilities only if $\tau>\tau_c$ (see Maksimov and Mints 1980). Hereafter we shall consider just the latter case. The extremely weak cooling ($W\ll\delta$) corresponds to quasiadiabatic thermal conditions. The results obtained in §3 are valid here in general for the $W/\delta\ll 1$ approximation. The qualitative behaviour of the curves $\lambda=\lambda(\beta)$ at various α is shown in figure 5(*a*) for $\tau<\tau_0$ and in figure 5(*b*) for $\tau>\tau_0$.

Let us consider first the case $\tau<\tau_0$. The dependence $\lambda=\lambda(\beta)$ is almost unchanged in the region $\lambda\gtrsim\lambda_m\gg 1$ with the increase of α (see figure 5(*a*), curves A–E). The function $\lambda=\lambda(\beta)$ changes more appreciably in the slow perturbation region ($\lambda i^2\ll 1$). The equation to determine the dependence $\lambda=\lambda(\beta)$ has the following form:

$$\lambda^2 D[1-(\alpha/\delta)](\tau-\tau_c)-\lambda[(\beta/\beta_0)-1]+\gamma W=0 \tag{34}$$

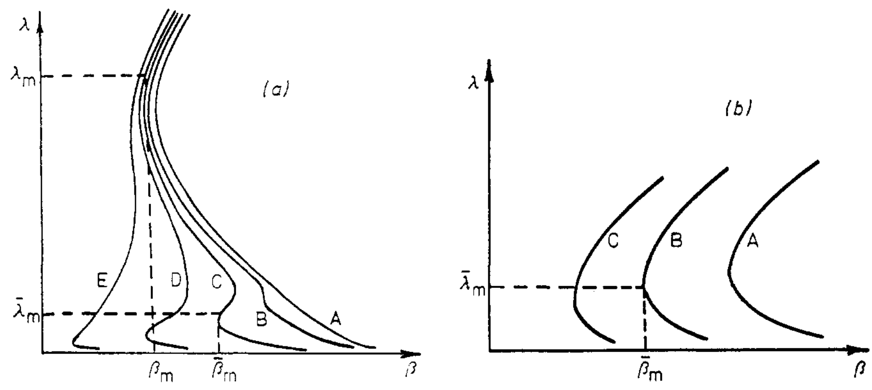

Figure 5. The function $\lambda=\lambda(\beta)$ for weak external cooling at different α. (*a*) quasi-adiabatical case ($W<\delta$), $\tau<\tau_0$; A, $\alpha<\alpha_1$; B, $\alpha\simeq\alpha_1$; C, $\alpha_1<\alpha<\alpha_2$; D, $\alpha=\alpha_2$; E, $\alpha_2<\alpha$. (*b*) $W\neq 0$, $\tau>\tau_0$.

where $\beta_0=\beta_0(\alpha)$ is defined by the expression

$$\beta_0=\beta_1\,[1-(\alpha/\delta)]$$

and the parameter τ_c is given by equation (18) or equation (33). For a flat sample

$$D(i)=0{\cdot}4\,i^2 \qquad \gamma=1 \qquad \beta_1=3/i^3.$$

For a wire

$$D(i)=B(i)/A(i) \qquad \gamma=2 \qquad \beta_1=1/A(i).$$

As seen from figure 5(*a*), the second minimum of the function $\beta=\beta(\lambda)$ appears if $\alpha\gtrsim\alpha_1$ (curve B). From equation (34) it is easy to determine the parameters $\bar{\lambda}_m$ (at which $(\mathrm{d}\beta/\mathrm{d}\lambda|_{\lambda=\bar{\lambda}_m}=0)$ and $\bar{\beta}_m=\beta(\bar{\lambda}_m)$ for $\alpha>\alpha_1$:

$$\left.\begin{aligned}\bar{\lambda}_m&=(\gamma W)^{1/2}\left[\left(1-\frac{\alpha}{\delta}\right)(\tau-\tau_0)\,D+\frac{\alpha}{\delta^2}\right]^{-1/2}\\ \bar{\beta}_m&=\beta_1\left(1-\frac{\alpha}{\delta}\right)\left\{1+2(\gamma W)^{1/2}\left[\left(1-\frac{\alpha}{\delta}\right)(\tau-\tau_0)\,D+\frac{\alpha}{\delta^2}\right]^{1/2}\right\}.\end{aligned}\right\} \tag{35}$$

This minimum becomes deeper (since $\bar{\beta}_m$ decreases) with the increase of α (curves C, D, E, figure 5(*a*)). At $\alpha<\alpha_2$ the relatively fast perturbations dominate in the vicinity of the stability threshold, i.e. $\lambda_c=\lambda_m\gg 1$ and $\beta_c=\beta_m(\alpha)$. At $\alpha>\alpha_2$ the stability is violated by slowly growing perturbations: $\beta_c=\bar{\beta}_m<\beta_m(\alpha)$ (figure 5(*a*), curve R). The criterion of thermomagnetomechanical instability is determined by the condition

$$\beta<\bar{\beta}_m$$

provided $\alpha>\alpha_2$. The dependence $\beta=\beta_c(\alpha)$ with the corresponding parameters is shown in figure 3(*a*).

For $\tau>\tau_0$ (figure 5(*b*)) the values $\bar{\lambda}_m$ and $\bar{\beta}_m$ are determined by expression (35) for all α. Figure 3(*b*) shows the function $\beta=\beta_c(\alpha)$.

Note that the results obtained in this subsection are also applicable in the case of adiabatic conditions on the sample surface, because there always exists some heat transfer (for example, through the sample holders) in the real experimental situation.

4.2. Dynamic stabilisation

In composite superconductors the critical state stability is ensured by heat transfer from the surface. The presence of the normal matrix damps the magnetic flux movement and at the same time it promotes effective heat removal into the coolant. Hence, slow flux jumps ($t_j > t_\kappa$) dominate in composite superconductors, strongly stimulating the plastic deformation jerks (Mints 1980). We shall consider this situation in more detail, i.e. we shall obtain the thermomagnetomechanical instability criterion for the conditions of dynamic stabilisation of the superconductor ($Wi^2\tau \gg 1$) under weak external cooling ($\delta \ll W \ll 1$).

It is obvious that $\tau \gg \tau_0$, if $i^2\tau \gg W^{-1} \gg 1$; hence figure 5(*b*) shows qualitatively the behaviour $\lambda(\beta)$. In the approximation $(i^2\tau)^{-1} \ll \lambda \ll W$ it is easy to obtain the dependence $\beta = \beta(\lambda)$ for the flat sample:

$$i\beta = (W-\alpha)\tau + \lambda\tau + \frac{(W-\alpha)}{i}\frac{\tau^{1/2}}{\lambda^{1/2}}. \tag{36}$$

From equation (36) it follows

$$\left.\begin{aligned} \lambda_c &= \left(\frac{W-\alpha}{2i}\right)^{2/3}\frac{1}{\tau^{1/3}} \\ i\beta_c &= (W-\alpha)\tau + 1{\cdot}9\,\frac{(W-\alpha)^{2/3}\tau^{2/3}}{i^{2/3}}. \end{aligned}\right\} \tag{37}$$

The relationship $\beta = \beta(\lambda)$ for the cylindrical sample is as follows:

$$i\beta = (2W-\alpha)\tau + \lambda\tau + 2\,\frac{(2W-\alpha)\tau^{1/2}(1-i)^{1/2}}{i\lambda^{1/2}}.$$

The analogous stability criterion for a wire with a fixed transport current has the form:

$$i\beta_c = (2W-\alpha)\tau + 3\left(\frac{(2W-\alpha)\tau}{i}\right)^{2/3}(1-i)^{1/3}. \tag{38}$$

The criterion (38) coincides with that obtained by Mints (1980), provided

$$(2W-\alpha)i^2\tau \gg 1 \tag{39}$$

Thus expression (39) is the condition of the applicability of the qualitative consideration. If the inequality (39) is violated with the increase of α, the dependence $\lambda = \lambda(\beta)$ can be found from the equation

$$\lambda^2 D(\tau-\tau_0) - \lambda[(\beta/\beta_1) - 1] + \gamma W - \alpha = 0 \tag{40}$$

(the parameters D, γ and β_1 are defined in §4.1). Equation (40) is relevant only if $0 < (\gamma W - \alpha)i^2\tau \ll 1$ and $W \ll 1$. From equation (40) one may obtain

$$\left.\begin{aligned} \lambda_c &= \left(\frac{\gamma W-\alpha}{D(\tau-\tau_0)}\right)^{1/2} \\ \beta_c &= \beta_1\{1 + 2[D(\gamma W-\alpha)(\tau-\tau_0)]^{1/2}\}. \end{aligned}\right\} \tag{41}$$

Equation (40) is invalid in the close vicinity of the plastic yield stability threshold (which can be seen from expression (41), in which β_c appears to have a complex value at $\alpha > \gamma W$). As shown by Mints and Petukhov (1980), the plastic flow instability occurs, if $\alpha > \alpha_c$, where α_c is

$$\alpha_c = \gamma W + 2(\gamma W\delta)^{1/2} + \delta. \tag{42}$$

The typical increment of the plastic deformation jerks is $\lambda_c \sim (W\delta)^{1/2} \gg \delta$. Therefore, the strain hardening should be taken into account, if $\alpha \to \alpha_c$. In the close vicinity of $\alpha = \alpha_c$, $(\gamma W < \alpha \leqslant \alpha_c)$, it is easy to find the dependence $\beta = \beta(\lambda)$:

$$(\beta/\beta_1) = 1 + (\gamma W/\lambda) - [\alpha/(\lambda+\delta)]. \tag{43}$$

From equation (43) it follows

$$\left.\begin{aligned} \lambda_c &= \delta\,[(\alpha/\gamma W)^{1/2} - 1]^{-1} \\ \beta_c &= \beta_1 \left\{1 - \frac{\gamma W}{\delta}\left[\left(\frac{\alpha}{\gamma W}\right)^{1/2} - 1\right]^2\right\}. \end{aligned}\right\} \tag{44}$$

The function $\alpha = \alpha_c(\beta)$ found from equation (44) is:

$$\alpha = \alpha_c(\beta) = \gamma W + 2(\gamma W\delta)^{1/2}[1 - (\beta/\beta_1)]^{1/2} + \delta[1 - (\beta/\beta_1)]. \tag{45}$$

Note that equation (45) is valid for an arbitrary $W \ll 1$. In particular, the function $\alpha = \alpha_c(\beta)$ obtained for the adiabatic case $W = 0$ (see the expressions (19) and (25)) follows from equation (45). Besides, the value $\alpha = \alpha_c(\beta = 0)$ from equation (45) coincides with the expression (42).

Finally, the dependence $\beta = \beta_c(\alpha)$ is determined by the following conditions:

$$\begin{aligned} &(\beta/\beta_3) + (\alpha/\gamma W) = 1 && (\gamma W - \alpha)i^2\tau \gg 1 \\ &(\beta/\beta_1) - 2[D(\gamma W - \alpha)(\tau - \tau_0)]^{1/2} = 1 && 1 \gg (\gamma W - \alpha)i^2\tau > 0 \\ &(\beta/\beta_1) + \frac{\gamma W}{\delta}\left[\left(\frac{\alpha}{\gamma W}\right)^{1/2} - 1\right]^2 = 1 && \alpha_c \geqslant \alpha > \gamma W \end{aligned} \tag{46}$$

where

$$\beta_3 = W\tau/i.$$

The first of the equations (46) was included in the main $(i^2\tau(\gamma W - \alpha) \gg 1)$ approximation. The function $\beta = \beta_c(\alpha)$ is shown in figure 3(*c*).

5. Discussion

A possible explanation of the training phenomenon can be suggested on the basis of the results obtained in the present work. As was shown above (§§3 and 4), the critical state becomes unstable with regard to the codeveloping flux jumps and plastic deformation jerks, if $\beta > \beta_c(\alpha, i, W, \delta, \tau)$. This inequality determines the range of the external parameters (e.g. the current i_q or the mechanical stress σ_q), at which the instabilities causing the normal quench occur. Each plastic deformation jerk is accompanied by strain hardening of the material, leading to a decrease of $\partial\dot{\epsilon}/\partial T$ (mainly due to the increase of $|\partial\dot{\epsilon}/\partial\epsilon|$), which results in an increase of the quench current in the next $(n+1)$th cycle:

$$(i_q)_{n+1} > (i_q)_n.$$

This successive quench current increase can take place in superconductors both for adiabatic thermal conditions (see expressions (19) and (25)) and for the conditions of dynamic stabilisation (see the criteria (37) and (38)). Thus the training phenomenon in superconductors can be explained as a consequent process of strain hardening, stimulated by heat softening of the material. The maximum attainable current $(i_q)_{max}$ is evidently determined by the limits of the mechanical hardening of the material. Naturally, the ratio $(i_q)_{max}/i_q(\sigma=0)$ represents the critical current degradation measure in the superconducting samples.

One should point out that the dependence of the quench current i_q on temperature is determined mainly by the functions $\nu(T)$ and $\beta(T)$ (for the adiabatically insulated samples) and by the function $\dot{\epsilon}(T)$ (in the presence of the weak external cooling).

Note that the sample temperature T_i is present in all stability criteria obtained (see expressions (19), (25), (37) and (38)). Therefore, to determine the direct relationship $i_q=i_q(T)$ (or, for example, $i_q=i_q(\sigma)$) the dependence of the temperature T_i on σ has to be taken into account.

It should be also mentioned that in the present work the stability threshold was obtained with respect to the uniform longitudinal (along z axis) perturbations. If the longitudinal nonuniformity is taken into account, it may considerably change the stability criteria obtained.

6. Conclusions

(i) The critical state stability has been considered taking into account the plastic yield of hard superconductors and superconducting composites.

(ii) The effect of strain hardening (§3) and external cooling (§4) on the stability criterion and the dynamics of thermomagnetomechanical instabilities have been investigated.

(iii) The results obtained are discussed in connection with the training phenomenon.

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