Dephasing Times in Closed Quantum Dots

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(Received 29 October 2001; published 14 March 2002)

Dephasing of one-particle states in closed quantum dots is analyzed within the framework of random matrix theory and the master equation. The combination of this analysis with recent experiments on the magnetoconductance allows, for the first time, the evaluation of the dephasing times of closed quantum dots. These dephasing times turn out to be dependent on the mean level spacing and significantly enhanced as compared with the case of open dots. Moreover, the experimental data available are consistent with the prediction that the dephasing of one-particle states in finite closed systems disappears at low enough energies and temperatures.

DOI: 10.1103/PhysRevLett.88.136801 PACS numbers: 73.23.Ad, 73.21.–b

Quantum coherence of electrons in closed quantum dots has attracted much interest in recent years [1–5]. Electron-electron interactions are believed to be one of the main sources of dephasing in disordered systems at low temperatures. Compared to low-dimensional metals and semiconductors [6], substantial modifications of this dephasing mechanism are caused by the confinement of the quantum dot which leads to discrete energy levels. In particular, the dephasing rate was predicted [2] to disappear at low excitation energies, $\epsilon < \Delta\sqrt{g/\ln g}$, where Δ is the mean level spacing and g is the dimensionless conductance of the dot.

Whereas there are a number of ways to measure the dephasing times in open quantum dots [7,8], the situation is much more complicated in closed dots. Only a few experiments have attempted to study dephasing in closed quantum dots. Most of these have focused on the relaxation of highly excited states, verifying the continuous to discrete spectrum transition at $\epsilon \propto g\Delta$ [3]. Some signatures of dephasing in thermalized states have been studied by Patel *et al.* [9], who analyzed the statistical distribution of the conductance maxima $G^{\max}$ (the height of the Coulomb blockade peaks). They found that the ratio of standard deviation to mean peak height $\sigma(G^{\max})/\langle G^{\max}\rangle$ is smaller than what random matrix theory (RMT) predicts [10], and attributed this reduction to dephasing effects. More recently, Folk *et al.* [4] suggested to use the dependence of the conductance upon applying a magnetic field B,

$$\alpha = (\langle G^{\max}\rangle_{B\neq 0} - \langle G^{\max}\rangle_{B=0})/\langle G^{\max}\rangle_{B\neq 0}, \quad (1)$$

as a probe of dephasing times. This is the closed dot analog of the weak localization magnetoconductance which was analyzed earlier for open dots [7]. RMT predicts $\alpha = 1/4$ [11,12], while Folk *et al.* measured considerably lower values of α, down to $\alpha \approx 0$ for the largest quantum dot with the maximal ratio k_BT/Δ (T is the temperature, k_B is the Boltzmann constant) interactions. Beenakker *et al.* [5] theoretically analyzed the situation in which the phase-breaking inelastic relaxation rate Γ_{in} [13] far exceeds the mean tunneling rate (inverse dwell time in the dot) $\overline{\Gamma}$. It turns out that, in this limit, α is reduced much stronger than found experimentally. Thus, they concluded that in the experiment [4] $\Gamma_{\text{in}} < \overline{\Gamma}$. However, as noted in Refs. [4,5], the lack of a quantitative theory of the crossover regime, $\Gamma_{\text{in}} \sim \overline{\Gamma}$, prevents a full analysis of the experimental results.

In this Letter, we study theoretically the effect of arbitrary inelastic scattering on the conductance of a closed quantum dot. We develop an analytical approach that allows one to evaluate α [Eq. (1)] and compare the results to the numerical solution. The approximate results are found to reasonably describe the behavior in the experimentally relevant temperature regime. Our calculations allow one, for the first time, to extract dephasing times of low lying (thermally excited) states in closed quantum dots. We observe a clear enhancement of the dephasing times relative to earlier results for open quantum dots [7]. Moreover, contrary to the analysis of open quantum dots [7] which showed a dependence on temperature alone, we find a dependence on *both* T and Δ. From our analysis it follows that the measurements of Folk *et al.* [4] are not inconsistent with a vanishing dephasing rate for low excitation energies [2]. A more detailed presentation of the calculation will be given in [14].

In the experimentally relevant regime $\hbar\Gamma_{\text{in}}, \hbar\overline{\Gamma} < k_BT, \Delta$, each state of the quantum dot is determined by a tuple $\{n_i\}$ of occupation numbers for the single particle eigenstates with energies E_i and spins S_i. The electrons can tunnel between the dot and the two leads. The left (L) and right (R) leads differ due to the applied voltage V. The elctrons in the leads are thermalized and distributed according to the Fermi function $f_{FD}(E) = [1 + e^{E/(k_BT)}]^{-1}$. The probability $P_{\mathcal{N}}(\{n_i\})$ to find a given set of occupation numbers $\{n_i\}$ with a total of $\mathcal{N}$ electrons (restricted to $\mathcal{N} \in \{N, N+1\}$ due to the Coulomb blockade) obeys the following master equation [15]:

$$\frac{dP_N(\{n_i\})}{dt} = \sum_{j\lambda} \delta_{n_j 0}\Gamma_j^\lambda[(1-f_j^\lambda)P_{N+1}(\{n_i\}_{+j}) - f_j^\lambda P_N(\{n_i\})] + \sum_{jk} \delta_{n_j 0}\delta_{n_k 1}[\Gamma_{\rm in}^{jk}P_N(\{n_i\}_{+j-k}) - \Gamma_{\rm in}^{kj}P_N(\{n_i\})],$$
$$\frac{dP_{N+1}(\{n_i\})}{dt} = \sum_{j\lambda} \delta_{n_j 1}\Gamma_j^\lambda[f_j^\lambda P_N(\{n_i\}_{-j}) - (1-f_j^\lambda)P_{N+1}(\{n_i\})] + \sum_{jk} \delta_{n_j 0}\delta_{n_k 1}[\Gamma_{\rm in}^{jk}P_{N+1}(\{n_i\}_{+j-k}) - \Gamma_{\rm in}^{kj}P_{N+1}(\{n_i\})]. \quad (2)$$

Here, $\{n_i\}_{+j}(\{n_i\}_{-j})$ are the tuples obtained from $\{n_i\}$ by adding (removing) one electron in the one-particle eigenstate j, and $f_j^\lambda = f_{FD}[E_j + (\delta_{\lambda L} - 1/2)eV - \mu]$, where $\lambda \in \{L, R\}$ and μ is the effective chemical potential, including the charging energy.

The first terms in (2) describe the tunneling of electrons between the dot and the leads. The additional terms $\Gamma_{\rm in}^{kj}$ in (2) describe inelastic scattering processes between the dot's one-particle eigenstates j and k. We assume that these are caused by thermal bosonic fluctuations at temperature T ($\omega_{jk} = E_j - E_k$),

$$\Gamma_{\rm in}^{jk} = \Gamma_{\rm in}^0 \frac{\mathrm{sgn}(\omega_{jk})D(|\omega_{jk}|)}{\exp[\omega_{jk}/(k_BT)] - 1}\,\delta_{S_jS_k}, \quad (3)$$

where $D(E)$ is the bosonic density of states. The form of Eq. (3) is very general; it assumes only detailed balance, no back coupling of the scattering to the Bose bath, and a spin-independent inelastic scattering. The microscopic mechanism might be due to external noise, electron-electron, or electron-phonon interaction. An important point with respect to Eq. (3) is that the suppression of α is quite robust to the specific model of interaction, and depends mainly on the *total inelastic scattering rate* $\Gamma_{\rm in}$. We consider $\Gamma_{\rm in}$ as a phenomenological parameter, to be determined experimentally. Since the experimental quantum dots [4] contain a large number of electrons, $N \gg 1$, they can be described by RMT [16]. In particular, the tunneling rates are Porter Thomas distributed $P_\beta(\Gamma) \propto \Gamma^{\beta/2-1}\exp[-\beta\Gamma/(2\overline{\Gamma})]$, with $\beta = 1, 2$ for the Gaussian orthogonal (no magnetic field) and unitary ensemble (with a magnetic field). The difference between $P_1(r)$ and $P_2(r)$ leads to the aforementioned, of $\alpha = 1/4$ in the absence of inelastic scattering [11,17].

The inelastic scattering model (3) is exponentially cut off to states outside an energy window of $\mathcal{O}(k_BT)$ and, thus, $\Gamma_{\rm in}$ vanishes at low temperatures. At $k_BT \gg \Delta$, on the other hand, there are many states $M \propto k_BT/\Delta$ connected by the inelastic scattering. Therefore, for $T \to \infty$, the total inelastic scattering rate $\Gamma_{\rm in}/\overline{\Gamma} \to \infty$ and the result of [5] is approached. In the leading order in Δ/k_BT, the solution of the master equation (2) is the equilibrium distribution and

$$G = \frac{e^2}{2k_BT}\frac{k_BT}{\Delta}\frac{\overline{\Gamma}^L\overline{\Gamma}^R}{\overline{\Gamma}^L + \overline{\Gamma}^R}, \quad (4)$$

i.e., $\alpha = 0$ (here, $\overline{\Gamma}^\lambda$ is the mean tunneling rate to lead λ, $\overline{\Gamma} = \overline{\Gamma}^L + \overline{\Gamma}^R$). Two corrections arise in the next order in Δ/k_BT: (i) the effect of a finite total inelastic scattering rate, and (ii) $\overline{\Gamma}$ is replaced by an average M levels around the Fermi energy, i.e., $\overline{\Gamma}^\lambda \to \langle\langle\Gamma_j^\lambda\rangle\rangle = \sum_j f_j(1-f_j)\Gamma_j^\lambda$ [5,15]. (Here and in the following, f_j is the Fermi function at *both* leads for $V = 0$.) This introduces corrections $\mathcal{O}(1/M)$ due to correlations between the numerator and denominator of Eq. (4). We calculate the former and take into account the latter by solving the master equation using perturbation theory in $\overline{\Gamma}/\Gamma_{\rm in}$, where $\Gamma_{\rm in}^j = \sum_{k\neq j}\Gamma_{\rm in}^{jk}(1-f_k)$. As a result [14],

$$G = \frac{e^2}{k_BT}P^{\rm eq}(N)\left(\frac{\langle\langle\Gamma^L\rangle\rangle\langle\langle\Gamma^R\rangle\rangle}{\langle\langle\Gamma^L + \Gamma^R\rangle\rangle} - \frac{\langle\langle\Gamma^{L2}\rangle\rangle\langle\langle\Gamma^R\rangle\rangle^2 + \langle\langle\Gamma^{R2}\rangle\rangle\langle\langle\Gamma^L\rangle\rangle^2 - 2\langle\langle\Gamma^L\rangle\rangle^2\langle\langle\Gamma^R\rangle\rangle^2}{\Gamma_{\rm in}^*\langle\langle\Gamma^L + \Gamma^R\rangle\rangle^2}\right), \quad (5)$$

$$\alpha = \frac{1}{12}\frac{\Delta}{k_BT} + \frac{\overline{\Gamma}}{2\Gamma_{\rm in}}, \quad (6)$$

where $P^{\rm eq}(N)$ is the equilibrium probability to have N electrons in the dot, and $\Gamma_{\rm in}^* = \Gamma_{\rm in}^j/(1-f_j)$. We neglect the weak j-dependence of $\Gamma_{\rm in}^*$. The total inelastic scattering rate $\Gamma_{\rm in}$ is defined as a value of $\Gamma_{\rm in}^j$ at Fermi energy, i.e., $\Gamma_{\rm in} = \Gamma_{\rm in}^j$ for $E_j = 0$.

In the following, we employ an approximation inspired by this high-T expansion. To first order in the voltage, one can write $P_{\mathcal{N}}(\{n_i\}) = P_{\mathcal{N}}^{\rm eq}(\{n_i\})[1 + \frac{eV}{k_BT}\sum_j \delta_{n_j1}\Psi(j)]$, where $P_{\mathcal{N}}^{\rm eq}(\{n_i\})$ is the equilibrium value of $P_{\mathcal{N}}(\{n_i\})$. With a large number of final states to scatter to, one can replace the sum over many individual terms $P(\{n_i\}_{+k-j})$ in the master equation (2) by an averaged quantity:

$$\sum_k \Gamma_{\rm in}^{kj}P_{N+1}(\{n_i\}_{+k-j}) - \Gamma_{\rm in}^{jk}P_{N+1}(\{n_i\}) = \frac{eV}{k_BT}P_{N+1}^{\rm eq}(\{n_i\})\sum_k \Gamma_{\rm in}^{jk}[\Psi(k) - \Psi(j)] \to \frac{eV}{k_BT}P_{N+1}^{\rm eq}(\{n_i\})\Gamma_{\rm in}[\bar\Psi - \Psi(j)]. \quad (7)$$

Here, $\bar\Psi$ should, in principle, be a weighted average over levels within a range of $\mathcal{O}(k_BT)$ around a particular level j considered. However, only levels around the Fermi energy are of interest for the conductance since the contribution of every level j to the conductance is multiplied by $f_j(1-f_j)$. For this reason, we approximately employ

a constant $\bar{\Psi} = \sum_j f_j(1-f_j)\Psi(j)/[\sum_j f_j(1-f_j)]$ in Eqs. (7) and (2) This leads to a self-consistent solution of the master equation with the result [14]

$$G = \frac{e^2}{k_B T} P^{\mathrm{eq}}(N) \left\langle\!\!\left\langle \Gamma_i^L \tau_i^{\mathrm{tot}} \left(\Gamma_i^R + \frac{\Gamma_{\mathrm{in}}^* \langle\langle \Gamma_j^R \tau_j^{\mathrm{tot}} \rangle\rangle}{\langle\langle (\Gamma_j^L + \Gamma_j^R)\tau_j^{\mathrm{tot}} \rangle\rangle} \right) \right\rangle\!\!\right\rangle, \tag{8}$$

where $\tau_i^{\mathrm{tot}} = (\Gamma_i^L + \Gamma_i^R + \Gamma_{\mathrm{in}}^*)^{-1}$. One would obtain the same form (8) but with $\langle\langle \cdots \rangle\rangle = \sum_{j=1}^{M} \ldots$ considering M degenerate levels filled with $\mathcal{N} \in \{0, 1\}$ electrons.

The result (8) can be interpreted in the following way: The first term represents processes in which the electron was not scattered at all. These happen with probability $(\Gamma_i^L + \Gamma_i^R)\tau_i^{\mathrm{tot}}$ and the resulting conductance peak heights are proportional to $(\Gamma_i^L \Gamma_i^R)/(\Gamma_i^L + \Gamma_i^R)$; yielding $\Gamma_i^L \Gamma_i^R \tau_i^{\mathrm{tot}}$ altogether. The second term represents contributions from electrons that were inelastically scattered after tunneling from one lead, and their contribution to the conductance is $\langle\langle \Gamma_j^R \tau_j^{\mathrm{tot}} \rangle\rangle / \langle\langle (\Gamma_j^L + \Gamma_j^R)\tau_j^{\mathrm{tot}} \rangle\rangle$.

Equation (8) is the main result of this paper. It is based on an approximation (7) which can be justified in the high-temperature limit. The particular advantage of this approach is that it gives not only the correct leading high-temperature behavior [Eq. (5)] but also reproduces correctly the limits $\Gamma_{\mathrm{in}} = 0$ and $\Gamma_{\mathrm{in}} = \infty$ for all T including $\alpha = 1/4$ at $T = 0$. Below, we demonstrate that this approach works pretty well in the intermediate regime $k_B T \sim \Delta$.

In order to calculate G and α, one has to average Eq. (8) with respect to the different ensembles. One could do so numerically, but it is possible to get analytical results via expanding Eq. (8) in powers of $\Delta/k_B T$ [14]. The first three terms in the $\Delta/k_B T$ expansion [or more specifically in $1/\sum_j f_j(1-f_j)$, which is $\propto \Delta/k_B T$ at large T] already give good accuracy in the relevant regime $k_B T > \Delta$ and are employed in the following. As we are interested in this regime, we assumed a picket fence distribution with spacing Δ between consecutive spin-degenerate levels ($E_{2j} = E_{2j-1} = j\Delta$; $\Gamma_{2j}^{\lambda} = \Gamma_{2j-1}^{\lambda}$; $\overline{\Gamma}_j^{\lambda} = \overline{\Gamma}/2$).

We tested the range of validity of this high-temperature approximation against the numerical solution [18] of the master equations (2). The latter is obtained by solving the master equation (2) by sparse matrix inversion [18]. Figure 1 compares values of α, as calculated using the first three terms in the $\Delta/k_B T$ expansion, with the numerical values. The agreement is very good for sufficiently high temperatures, and reasonable even for low T. In the whole temperature regime, the deviations are within current experimental accuracy.

It, thus, appears that our analytical approach provides a reliable way to determine Γ_{in} from the experimental measurements of α, in the whole temperature regime. For future experiments we provide Fig. 2, which presents α as a contour plot in the space spanned by $k_B T/\Delta$ and $\Gamma_{\mathrm{in}}/\overline{\Gamma}$.

A direct experimental test would be provided by measuring values of α in a given dot at fixed T, as a function of $\overline{\Gamma}$ (which can be achieved by changing the contact setting). The theoretical dependence of α on $\overline{\Gamma}$ involves a single fitting parameter, i.e., the unknown total scattering rate Γ_{in} which is assumed to be unaffected by the contact setting. A first step in this direction was done in [4], and in the inset of Fig. 3 we compare the prediction of our high-temperature approximation with the measurements of α for three different values of $\overline{\Gamma}$. An excellent agreement is obtained, though more data points are required for reliable conclusions.

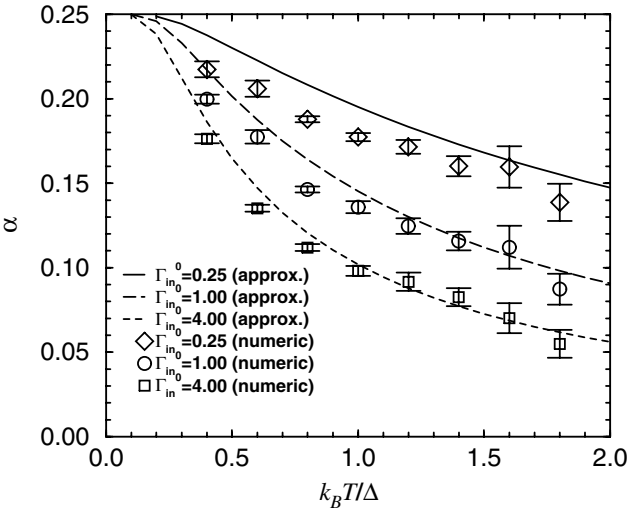

FIG. 1. Comparison of the numerical solution of the full master equation with the high-temperature approximation. The latter is seen to work well for $k_B T > \Delta$.

We now use the above theory to extract dephasing times from the data points (mean values and error bars) of Folk *et al.* [4]. Figure 3 presents these estimates as symbols and error bars, respectively, and compares them with open

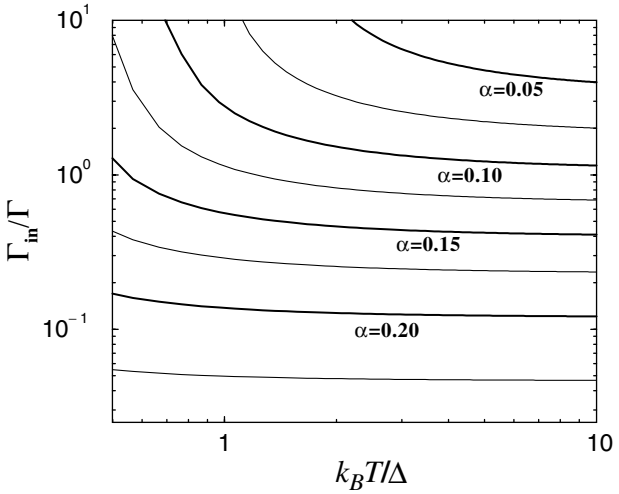

FIG. 2. A contour plot of α as a function of $\Delta/k_B T$ and $\Gamma_{\mathrm{in}}/\overline{\Gamma}$, based on the high-temperature approximation. The values of the bold contours are specified. Given T, Δ, and α from future experiments, one can extract $\Gamma_{\mathrm{in}}/\overline{\Gamma}$ from this figure.

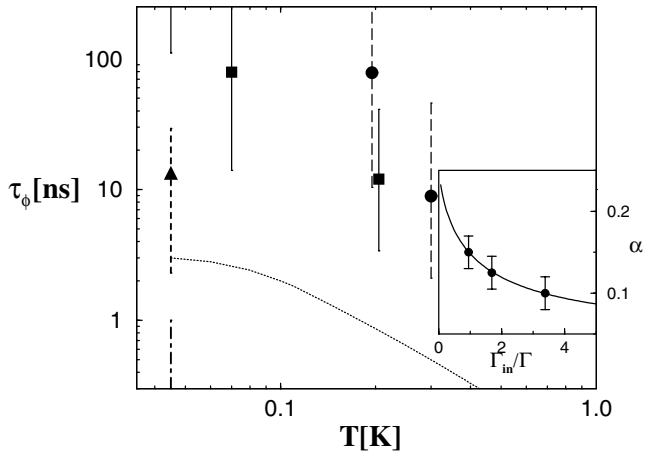

FIG. 3. Dephasing times, τ_ϕ, as extracted from the data points in Ref. [4] for four different dots: $\Delta = 28$ μeV (circles, long-dashed error bars), $\Delta = 10$ μeV (squares, solid error bars), $\Delta = 2.4$ μeV (up-triangles, dashed error bar), and $\Delta = 0.9$ μeV (dot-dashed error bar); dotted line: fit to open dot experiments as calculated in [7]. Error bars which extend up (down) beyond the graph should be understood as going up to infinity (down to zero); if no corresponding point is visible, the experimental mean value itself gives $\tau_\phi = \infty$ (or $\tau_\phi = 0$). In the inset, we fit experimental measurements for different values of $\overline{\Gamma}$ [4] with our theory. The single fitting parameter is $\hbar\Gamma_{\rm in} = 0.25$ μeV, or $\tau_\phi = 16$ ns.

dot values [7]. A clear enhancement of the dephasing times compared to open dots is observed. In addition, dephasing times strongly depend on Δ (as can be seen at $T = 45$ mK). This is in contrast to open dot results [7]. An additional suppression of α for $k_BT < \Delta$, resulting from level-spacing fluctuations [14,17], was not included in our analysis, and therefore our results *underestimate* the dephasing times for $k_BT < \Delta$. In addition, the result for the $\Delta = 0.9$ μeV quantum dot, which is consistent with $\tau_\phi = 0$, should be interepreted carefully since the result implies $\hbar\Gamma_{\rm in} > \Delta$ and the master equation is not applicable anymore. Based on our analysis, the recent experiment [4], measuring dephasing in closed quantum dots is consistent with dephasing due to electron-electron interaction alone, including the prediction of the critical vanishing of dephasing rate. However, given the large error bars of the current experimental data, one cannot exclude an algebraic behavior or even a saturation of the dephasing rates for $T \to 0$. Nevertheless, the behavior is clearly different from that of open quantum dots [7] and is Δ dependent.

In conclusion, we provide a theoretical approach to extract the inelastic scattering rate in closed dots based on measurements of the weak-localization correction α. Analyzing a recent experiment by Folk *et al.* [4], we see a clear enhancement of the dephasing time compared with open dot values. There is no inconsistency with theoretical predictions for electron-electron interaction, in particular, a vanishing dephasing rate at a critical Δ-*dependent* temperature. We note, however, that the available experimental data is limited and has considerably statistical uncertainties. Future experiments are necessary, and we offer Fig. 2 to extract the temperature and level-spacing dependence of the inelastic scattering rate and to thoroughly test the prediction of a diverging dephasing time.

We are happy to acknowledge intensive and very helpful discussions with Igor Aleiner, Joshua Folk, and Charles Marcus. This work has been supported by ARO, DARPA, and the Alexander von Humboldt foundation.

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[18] For $k_BT < 1.6\Delta$, we took into account all configurations $\{n_i\}$ which involve levels in $[-4k_BT, \ldots, 4k_BT]$ with $P^{\rm eq}\{n_i\}/\min_{\{n_i\}} P^{\rm eq}\{n_i\} > \exp(-5\Delta/k_BT)$; for $k_BT > 1.6\Delta$, the interval was $[-3k_BT, \ldots, 3k_BT]$ and the exponential cutoff $-4.5\Delta/k_BT$ (see [14] for details).