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Metastability in the random-field Ising model

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Effects of metastability in random-field Ising systems are calculated for domains that are both curved and rough. Villian's and Bruinsma and Aeppli's scaling forms for the domain size are obtained from the same approach and the crossover between them is simply explained. Generalizations to random fields with nonzero averages lead to a "freezing line" and are relevant to experiments on binary-fluid mixtures in gels and in porous media.

Much controversy has existed in recent years about the behavior of the random-field Ising model (RFIM). Theoretically, conflicting results were obtained for the value of the lower critical dimension d_l of this model. This is the dimension below which there is no long-range order even in the presence of a vanishing small random field. Some works gave $d_l = 2$,[1-5] whereas others gave $d_l = 3$.[6] The consensus today is that most probably the lower critical dimension of the RFIM is two, $d_l = 2$, which agrees with the original and simple domain argument of Imry and Ma.[1] This result is supported by a rigorous proof at zero temperature,[5] by other works that carefully take into account domain-wall roughening,[2] and by numerical investigations on finite-size lattices.[3,4] Possible faults in all present theories that predict $d_l = 3$ are very nicely summarized in Ref. 7.

The fact that at thermodynamical equilibrium there is order in $d = 3$ (since most probably $d_l = 2$), does not agree, at first sight, with neutron scattering experiments[8,9] that are done on diluted antiferromagnets in the presence of an external magnetic field.[10] In these experiments,[8] as one cools down the sample from high temperatures keeping the magnetic field at a nonzero value *(field cooling)*, domains (sometimes very large) are observed even at very low temperatures; hence a lack of long-range order is observed in $d = 3$. The same system *does* show long-range order after a *zero-field cooling*, where the sample is cooled in zero external field and the field is then turned on.

A very likely explanation for these experiments can follow if one distinguishes between the long-range ordered state which is the equilibrium state of the $d = 3$ system at low enough temperatures and random fields, and metastable states in which domain walls are pinned by the random field. As one field cools, the system can be trapped in one of these metastable states even though the lowest-energy state has long-range order. A theory that introduced these ideas and calculated the scaling behavior of the domain size R as function of the random field H was developed recently by Villain.[11] Close to the transition temperature $R \sim H^{-\nu_H}$ with $\nu_H = 2/(2-\eta)$, η being one of the usual critical indices, whereas at low temperatures $R \sim H^{-\nu_H}$ with $\nu_H = 2$. These results for ν_H do not depend explicitly on the system dimensionality d and hold for $d > 2$ (domain growth and metastability were also investigated numerically).[12]

Somewhat simultaneously and quite independently other theories that also considered metastability were published. Grinstein and Fernandez[13] arrived at the same result for $\nu_H = 2$ for low temperatures, whereas Bruinsma and Aeppli[14] arrived at a completely different scaling form for $R(H)$. According to their theory $R \sim H^{-\nu_H}$ with $\nu_H = 4/(5-d)$, which accidentally gives $\nu_H = 2$ for $d = 3$ but does not agree with Villain's result for other dimensionalities. Moreover, even for $d = 3$ the temperature dependence of the prefactor in the proportionality $R \sim H^{-\nu_H}$ differs for the two theories.

In this paper, we propose an explanation for the apparent disagreement between the two theories.[11,14] We rederive Villian's results using a somewhat different formulation. We also obtain, from the same theory, Bruinsma and Aeppli's result as a special limit which holds for very low temperatures. To our knowledge, this is the first unified explanation of the two formulas for $R(H)$, since they were originally obtained following a completely different line of reasoning. We also calculate the effect of a constant magnetic field h on metastability in the RFIM. Hysteresis and transitions between metastable states and the equilibrium state are found in the h-T plane (keeping H fixed). These predictions are relevant to systems of binary-fluid mixtures in gels or in porous media.[15]

Our starting point is to write the energy barrier between two metastable states of the system.[7,11] We look on a single domain of "down" spins with a well-defined radius R embedded in a region of "up" spins. At equilibrium this domain is unstable in $d = 3$, i.e., it collapses. But due to the effects of random-field pinning, the shrinking of the domain is carried out by jumping over energy barriers that exist between "successive" metastable states.[11,13,14] Each one of these states is characterized by an average displacement w, which is a measure of the roughness over a length scale b (see Fig. 1). It is important to note that the displacement is measured with respect to the smooth interface that has a curvature R. The energy barrier between such two states can be written as

$$\Delta F(w,b) = -g\left[b^{d-1}\left(\frac{w}{b}\right)^2 + b^{d-1}\left(\frac{w}{R}\right)\right] + Hm_0\,(b^{d-1}w)^{1/2}\,, \tag{1}$$

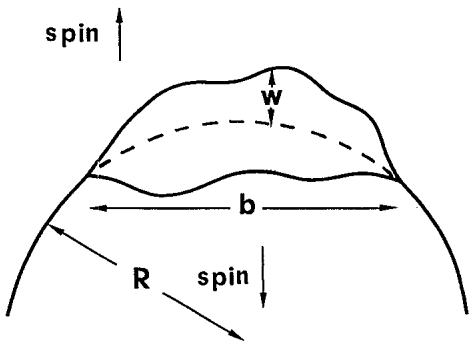

FIG. 1. A domain (curvature R) of "down" spins embedded in a region of "up" spins. On a length scale b, there are two possible metastable states of the interface, each with average displacement w.

g being the interfacial tension, H the strength of the random field, and $-m_0$ ($+m_o$) the average magnetization inside (outside) the domain. The first term in Eq. (1) is the interfacial energy for the increase of area of a continuous interface due to roughness (displacement w); the second term is the Laplace contribution for a curved interface of radius R; the third term is the usual magnetic fluctuation term that scales as the square root of the difference in volume between the two states. Notice that when $R \to \infty$, namely, the interface becomes flat but remains rough, we get back the same expression that was used in Refs. 2 and 7 to prove that roughness does not modify the Imry-Ma result[1] and that $d_l = 2$. In Eq. (1), w and b are two independent variables; maximizing with respect to w we will get the maximum energy barrier for each length scale b. It is easy to obtain this result in two limits:

(a) When $w/R << (w/b)^2$, neglecting the second term of Eq. (1) gives us the flat-interface results

$$\Delta F_{\max}(b) = g\left(\frac{Hm_0}{g}\right)^{4/3} b^{(d+1)/3} , \tag{2}$$

and

$$w_{\max}(b) = \left(\frac{Hm_0}{g}\right)^{2/3} b^{(5-d)/3} . \tag{3}$$

(b) When $w/R >> (w/b)^2$, neglecting the first term in Eq. (1) we get

$$\Delta F_{\max}(b) = Rg\left(\frac{Hm_0}{g}\right)^2 , \text{ independent of } b , \tag{4}$$

and

$$w_{\max}(b) = \left(\frac{Hm_0R}{g}\right)^2 b^{1-d} . \tag{5}$$

A schematical plot of $\Delta F_{\max}$ and $w_{\max}$ as functions of b is shown in Fig. 2. The crossover between cases (a) and (b) occurs at b^* such that $b^* = (w_{\max}R)^{1/2}$. Using the value of $w_{\max}$ from Eq. (3) or equivalently from Eq. (5) we get for b^*

$$b^* = R^{3/2(d+1)}\left(\frac{Hm_0}{g}\right)^{2/(d+1)} . \tag{6}$$

For $b < b^*$ both $w_{\max}$ and $\Delta F_{\max}$ are monotonically increasing functions of b, whereas for $b > b^*$, $\Delta F_{\max}$ is b independent and $w_{\max}$ is decreasing. The radius of the minimal metastable domain is easily determined by comparing the maximal energy barrier $\Delta F_{\max}(b)$ with the thermal fluctuation k_BT. For a small domain, $\Delta F_{\max}(b) < k_BT$ for all b, and the domain collapses since all barriers are smaller than the thermal fluctuaion. For large domains, k_BT is smaller than barriers on large length scales b, but is bigger than the smaller barriers as is shown in Fig. 2. Hence, the minimal domain size $R_{\min}$ is determined from $k_BT = \Delta F_{\max}(b^*)$, $\Delta F_{\max}(b^*)$ being taken from Eq. (4). This leads to Villain's result

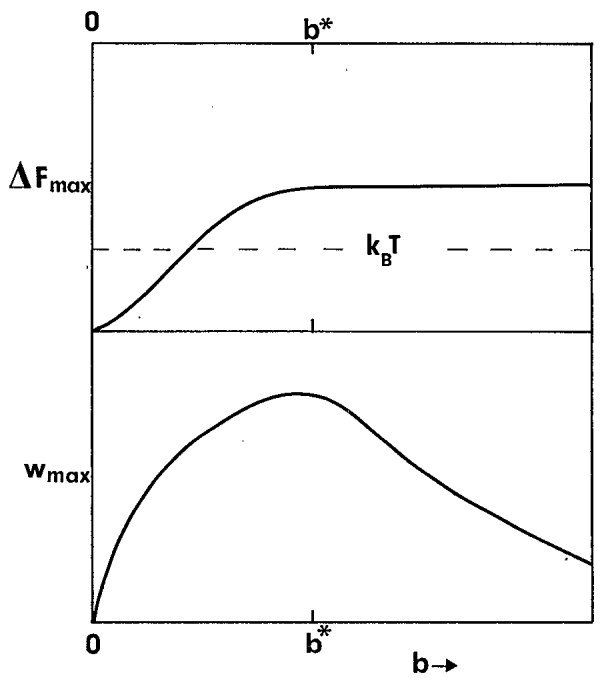

FIG. 2. A schematic plot of the maximum energy barrier $\Delta F_{\max}$ and its corresponding displacemnt $w_{\max}$ as functions of the length scale b. b^* is equal to $(w_{\max}R)^{1/2}$. The dashed horizontal line is a thermal fluctuation k_BT.

$$R_{\min} = gk_BT(Hm_0)^{-2} . \tag{7}$$

The exponent ν_H can be calculated in two cases:

(i) For low temperatures $m_0 \sim 1$ and $g \simeq J/\xi^{d-1} \sim J/a^{d-1}$ (J is the exchange interaction and the correlation length ξ is approximately equal to the spin separation a). For this case $R_{\min} \sim TH^{-\nu_H}$ with $\nu_H = 2$.

(ii) For temperatures close to the phase-transition temperature T_c, using the same scaling laws for m_0 and $g \simeq J/\xi^{d-1}$ as for the nonrandom case $m_0 \sim \xi^{-\nu/\beta}$ and $g \sim Jm_0^{(d-1)\nu/\beta}$, we get $R_{\min} \sim k_BTJH^{-2}m_0^{-2(\eta-1)/(d-2+\eta)}$ [using the scaling law $\nu/\beta = 2/(d-2+\eta)$]. Since the correlation length ξ is always assumed to be smaller than $R_{\min}$, this picture of metastable domains breaks down when $R_{\min} \simeq \xi \sim m_0^{-\nu/\beta}$. Thus, close to T_c, the domain size scales with the random field with an exponent $\nu_H = 2/(2-\eta) \simeq 1.0$.[11]

We now proceed to show why at very low temperatures, $R(H)$ is not equal to $R_{\min}$, but rather is the domain size as was found in Ref. 14. In obtaining $R_{\min}$, Eq. (7), the interface was assumed to be continuous and well defined. This

is true as long as w, the interface displacement, is bigger than ξ, the width of the interface (ξ is also the bulk correlation length). For large domains, $w_{\max}(b) > \xi$ at least in a range of length scales close to b^*. As the temperature decreases, $R_{\min}$ decreases and so does $w_{\max}(b^*)$, until at some temperature $w_{\max}(b^*) = \xi$. This now defines a new domain size R_ξ [Eq. (5)],

$$w_{\max}(b^*) = (Hm_0R_\xi)^2 g^{-2}(b^*)^{1-d} \simeq \xi \,, \tag{8}$$

so that

$$R_\xi = \xi^{(d+1)/(5-d)} \left(\frac{Hm_0}{g} \right)^{-4/(5-d)} \,. \tag{9}$$

Equation (9) is exactly the result of Bruinsma and Aeppli.[14] In what follows we look at $d=3$, where $R_\xi = \xi^2 \times (Hm_0/g)^{-2} = (Hm_0/J)^{-2}$. Hence, the scaling of R_ξ with H is (accidentally) the same as in Eq. (7), but the prefactor is different.

The actual domain size $R(H)$ as function of H and T can be determined from the comparison between the two domain sizes $R_{\min}$ and R_ξ. As long as $R_{\min} > R_\xi$ the domain boundary is well defined and $R(H) = R_{\min}$. In this case the maximal energy barriers determine the minimal size of metastable domains. This is correct down to temperatures where $R_{\min} = R_\xi$, or, equivalently $k_B T = J$. Using the mean-field value of T_c we get that this temperaure $T^* = T_c/6$, where $2dJ = k_B T_c$. For temperatures lower than T^*, $R_\xi > R_{\min}$ and the actual domain size will be $R(H) = R_\xi$. The reason for this is that for $R < R_\xi$ there are no energy barriers at all. Figure 3 summarizes our predictions for $R(H)$. For $T < T^* \simeq T_c/6$, $R(H) = R_\xi = (H/J)^{-2}$, which is temperature independent. For $T > T^*$, $R(H) = R_{\min}$ will grow linearly with T and closer to T_c it will grow much faster until $R(H) \simeq \xi$, where the domain picture breaks down (Fig. 3). An additional remark is that the crossover between $R_{\min}$ and R_ξ is independent of H only in $d=3$, where ν_H of R_ξ is also 2. For $d=2$, ν_H of R_ξ is $\frac{4}{3}$, whereas $\nu_H = 2$ for $R_{\min}$. This implies a more complicated crossover in the T-H plane for $d=2$. Our theory assumes that the temperature is quenched rapidly from high temperatures (so do Refs. 11–14), and also that there is no relaxation out of metastable domains with barriers higher than $k_B T$. However, very slow (logarithmic) relaxation with time is expected.[11,13]

Constructing a simple theory that takes into account roughness and curvature effects of domains, we obtain both the results of Refs. 11 and 14. In three dimensions the crossover between the two scaling forms of $R(H)$ depends only upon the temperature and the result of Ref. 14 holds only for very low temperature. We apply the same ideas also to explain other experimental realizations of the RFIM which are binary mixtures in gels or in porous media.[15] Details will be given elsewhere[16] and here we state one of our main findings which is also relevant for the RFIM. When a constant magnetic field h is added on top of the random field H, the domain size $R_{\min}(H,h)$ is related to $R_{\min}(H, h=0)$ in the following way:

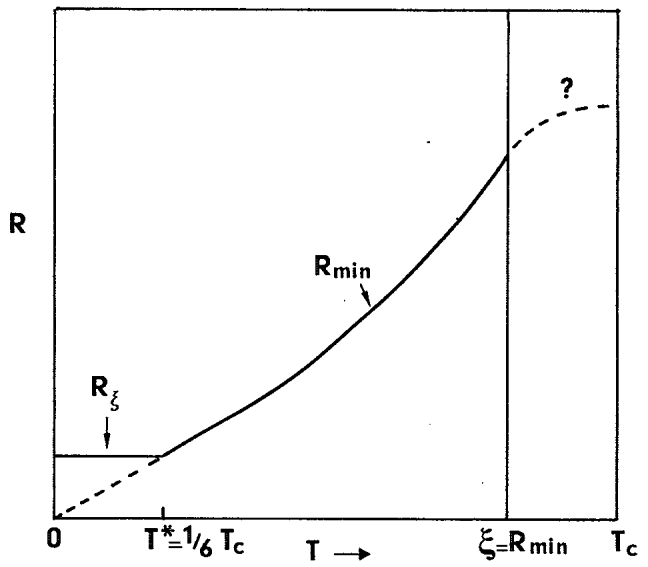

FIG. 3. A schematic plot of the domain size R as function of temperature T for a given random field H. The crossover between R_ξ and $R_{\min}$ occurs at $T = T^*$. The theory is consistent up to a temperature where $R_{\min} \simeq \xi$. Above this temperature we expect a saturation of R.

$$R_{\min}^{-1}(H,h) = R_{\min}^{-1}(H,h=0) - hm_0/g$$
$$= (Hm_0)^2/gk_BT - hm_0/g \,. \tag{10}$$

For a fixed H, there is a line in the T-h plane for which $R_{\min}(H,h) \to \infty$. This line $k_B T_g = (Hm_0)^2/hm_0$ is a "freezing line" separating the metastable region from the equilibrium region.[16] In concept it is similar to the de Almeida–Thouless line[17] for spin glasses. We also find[16] a hysteresis in the T-h plane depending on whether one cools in constant field $h > 0$ or one cools with $h = 0$ and then applies a constant field. Even though in the magnetic systems h corresponds to a staggered field which is not realizable, for the gel plus binary mixture systems, our results could be checked by putting the gel in contact with a reservoir of the binary-fluid mixture, thus creating a difference in chemical potential which acts as a constant magnetic field.

After this work was completed, we learned that T. Nattermann has obtained similar results about the relation between Refs. 11 and 14, using a different approach.

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