Effect of spectral fluctuations on conductance-peak height statistics in quantum dots

K. Held, E. Eisenberg, and B. L. Altshuler
Physics Department, Princeton University, Princeton, New Jersey 08544
and NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540
(Received 4 March 2002; published 29 July 2002)

Within random matrix theory for quantum dots, both the dot's one-particle eigenlevels and the dot-lead couplings are statistically distributed. While the effect of the latter on the conductance is obvious and has been taken into account in the literature, the statistical distribution of the one-particle eigenlevels is generally replaced by a picket-fence spectrum. Here we take the random matrix theory eigenlevel distribution explicitly into account, and observe significant deviations in the conductance distribution and magnetoconductance of closed quantum dots at experimentally relevant temperatures.

DOI: 10.1103/PhysRevB.66.033308 PACS number(s): 73.23.Ad

The universal statistical fluctuations observed at the low-energy part of the spectrum of quantum systems whose associated classical dynamics are chaotic can be described by random matrix theory (RMT). This type of description can be justified for diffusive quantum dots and quantum dots with irregular shapes,[1,2] which makes quantum dots a particular example for the study of RMT fluctuations. While in open quantum dots (which have a strong dot-lead coupling) the effect of the electron-electron interaction is mostly neglected, this interaction leads to the Coulomb blockade in closed quantum dots (see Ref. 3 for a review): The low-temperature conductance is heavily suppressed due to the large charging energy required to add an electron to the quantum dot, except for the Coulomb blockade peaks at which the potential of the quantum dot is adjusted such that N and $N+1$ electrons in the dot have the same energy. The RMT approach[1,4] successfully describes the mesoscopic fluctuations of these Coulomb blockade peaks, i.e., the statistical distribution of their height $P(G^{\max})$ and its dependence upon a magnetic field.[5] On the other hand, recent improved experiments showed significant deviations from the RMT prediction, suggesting that interaction effects beyond charging should be considered as well. In particular, dephasing of single-particle states due to interactions modifies the conductance peak height statistics (see Refs. 1 and 2, and references therein). In a recent experiment, Patel *et al.*[6] found that the statistical distribution has a smaller ratio of standard deviation to mean peak height $\sigma(G^{\max})/\langle G^{\max}\rangle$ than predicted by RMT,[7] and attributed this to dephasing effects. In another experiment, Folk *et al.*[8] measured the change of the conductance in a magnetic field B,

$$\alpha=\frac{\langle G^{\max}\rangle_{B\neq 0}-\langle G^{\max}\rangle_{B=0}}{\langle G^{\max}\rangle_{B\neq 0}}, \tag{1}$$

as a probe of dephasing times. This is the closed dot analog of the weak-localization magnetoconductance which had proven to be an effective measure for open dots dephasing times.[9] It was pointed out that $\alpha=1/4$ as long as the transport is dominated by elastic scattering.[7,10] Therefore, any deviation of the measured α from 1/4 was considered an indication for dephasing.

In this Brief Report, we discuss the effects of spectral fluctuations of the RMT one-particle eigenlevels on the statistical distribution $P(G^{\max})$ and the weak-localization correction α. Previous works[1,4,7,10,11] generally considered a picket-fence spectrum, i.e., a rigid level spacing between successive eigenlevels in the quantum dot, for the calculation of the conductance. This ignores the effect of spectral eigenlevel fluctuations. The picket-fence spectrum is a good approximation for both very high temperatures and very low temperatures,[1] and a comparison of $P(G^{\max})$ with full RMT statistics and a picket-fence spectrum without spin degeneracy at three temperatures showed only minor deviations.[12]

In the Brief Report, we study the full RMT statistics in detail with and without spin-degeneracy, and find significant differences compared to the picket-fence spectrum, in particular in an experimentally relevant regime $k_BT\lesssim\Delta$. The spectral fluctuations lead to lower values of α than 1/4, such that this value is not universal, even in the absence of any dephasing mechanism. One therefore has to be careful while using α as a probe for dephasing in this temperature regime.

Within the constant interaction model, the conductance of a quantum dot is given by the formula[13]

$$G=\frac{e^2}{kT}\sum_{i,N=1}^{\infty}\frac{\Gamma_i^L\Gamma_i^R}{\Gamma_i^L+\Gamma_i^R}P_{\rm eq}(N)P(E_i|N)[1-f(E_i-\mu)], \tag{2}$$

where $\Gamma_i^{L(R)}$ is the tunneling rate between the ith one-particle eigenlevel of the dot and the left (right) lead, $P_{\rm eq}(N)$ is the equilibrium probability of finding N electrons in the dot with the Coulomb blockade allowing for N and $N+1$ electrons, $P(E_i|N)$ is the canonical probability to have the level i occupied given the presence of N electrons in the dot, and $f(E)$ is the Fermi function at the effective chemical potential μ, which includes the charging energy. In a typical experimental situation, the charging energy is much larger than the temperature, and thus only one term contributes to the sum over N. In Eq. (2), $\Gamma_i^{L(R)}$ is Porter-Thomas distributed in the Gaussian orthogonal ensemble (GOE) and Gaussian unitary ensemble (GUE) without and with a magnetic field, respectively, and the eigenlevel energies E_i obey the respective RMT distribution.[1] In contrast, the picket-fence spectrum has $E_{2i}=E_{2i-1}=i\Delta$ in the case of spin degeneracy and $E_i=i\Delta/2$ without spin degeneracy. The first term in the sum $\Gamma_i^L\Gamma_i^R/(\Gamma_i^L+\Gamma_i^R)$ depends only on the eigenfunctions of the dot, and thus is uncorrelated with the spectrum within the RMT approach. The ensemble average of this term in the absence (GOE) or presence (GUE) of a magnetic field is

$$\left\langle\left\langle \frac{\Gamma_i^L\Gamma_i^R}{\Gamma_i^L+\Gamma_i^R}\right\rangle\right\rangle=\begin{cases}1/4 & \text{GOE}\\ 1/3 & \text{GUE.}\end{cases} \tag{3}$$

This yields the value $\alpha=1/4$ if the weights $P(E_i|N)$ are the same for both ensembles. This should be the case in the low-temperature regime $k_BT\ll\Delta$, since only one level E_0 contributes with maximal weight: $P(E_i|N)\approx\delta_{i0}$. In general, the main contribution to the sum comes from $O(k_BT/\Delta)$ levels around the Fermi energy which gives the same contribution at large temperatures $k_BT\gg\Delta$ for the GOE and GUE, implying $\alpha=1/4$ in this regime as well.

However, for $k_BT\lesssim\Delta$, the probability to have more than one level in an energy window k_BT around the Fermi energy is increased for the RMT eigenlevel distribution compared to the picket-fence spectrum. These additional levels enhance the conductance. Since there are more nearby levels for the GOE case, due to the weaker level repulsion, the GOE conductance is enhanced more, and α is suppressed.

A second important effect is the optimization of the chemical potential for the Coulomb blockade peak. This effect was generally ignored, as it is technically cumbersome to consider, and is not significant for both very low and very high temperatures. Disregarding this effect means that a theorist optimized the chemical potential with respect to the averaged conductance, instead of optimizing for every realization as in the experiment. Whenever there is a nearby level, the position of the peak is shifted to optimize the contribution from both levels. Typically, a level with very low tunneling rates (and, thus, a suppressed conductance peak) would be significantly enhanced by contributions from its neighbors. If the tunneling rate of a neighboring level is much higher, the peak position $\mu^{\max}$ is shifted toward it. As the distribution of level spacings is different depending on the existence of a magnetic field, this enhancement mechanism is again more effective in the absence of a magnetic field (GOE), where probabilities of small spacing and of small conductances are higher. Thus this effect, which was neglected in Ref. 12, suppresses α even further. While these eigenlevel fluctuations lead to a suppression of α, another mechanism, i.e., the exchange interaction not considered in the present paper, tends to increase α.[14]

We evaluated the sum (2) numerically by drawing $\Gamma_i^{L(R)}$ from the Porter-Thomas distribution and E_i according to the Wigner-Dyson distribution. Levels within a window of $\pm 4k_BT$ around the Fermi energy have been taken into account and the Fermi energy μ in Eq. (2) has been adjusted to yield $G^{\max}$ for every realization.

Figure 1 compares the probability distribution $P(G^{\max})$ for a picket-fence spectrum vs. the full RMT level statistics. As explained above, RMT spectral fluctuations enhance the conductance. In particular, the probability to have a very low $G^{\max}$ is reduced and the probability to have an intermediate $G^{\max}$ is enhanced. The reason for the reduction is that a very low $G^{\max}$ requires Γ^L or Γ^R in Eq. (2) to be low. RMT spectral fluctuations enhance the contributions from nearby levels, which typically do not have a low value of $\Gamma^{L(R)}$ at the same time. Thus the peak position of μ is shifted toward a nearby level, and the conductance occurs through both levels. Notably, the effect of phase-breaking inelastic scattering processes leads to similar changes.[11]

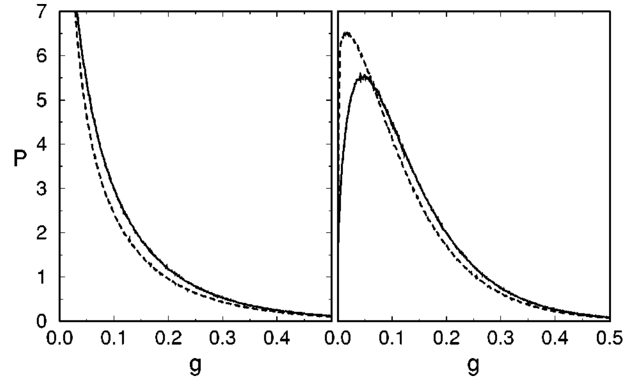

FIG. 1. Probability distribution $P(g)$ of the dimensionless closed dot conductance g defined by $G^{\max}=(e^2/\hbar)(\hbar\overline{\Gamma}/k_BT)g$ at $k_BT=0.2\Delta$ in the presence of spin degeneracy (left: GOE; right: GUE; solid line: RMT spectral fluctuations; dashed line: picket fence).

Deviations of α from the "universal" value 1/4 have been interpreted as being a result of dephasing. While dephasing would certainly suppress α, we note here that in the regime $k_BT\lesssim\Delta$ the effects of spectral fluctuation discussed above, lead to a similar effect. In Fig. 2 we present results for α as a function of the scaled temperature k_BT/Δ, for both, the spin-degenerate spectrum and the case of broken symmetry. While the effect seems to be small, one should keep in mind that, in the low-temperature regime, even very strong

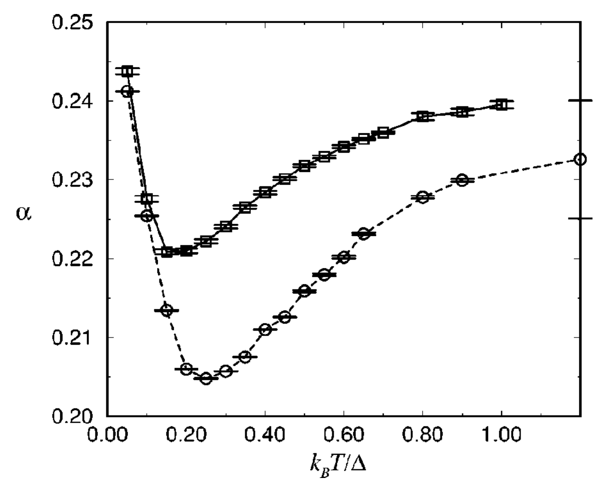

FIG. 2. Magnetoconductance α vs k_BT/Δ for the spin-degenerate case (dashed line) and without spin degeneracy (solid line). Taking into account the RMT spectral fluctuations, α is reduced from its "universal" value $\alpha=1/4$, in particular in the experimentally relevant regime $0.1\Delta<k_BT<0.8\Delta$.

dephasing does not suppress α to zero,[10] and thus the correction due to spectral fluctuations is comparable with or even larger than the effect of dephasing.[10,11] One should therefore cautiously use α as a probe of dephasing in this regime. In particular, the suppression of α below 1/4 observed in Ref. 8 at $k_B T < \Delta$ can be explained by the effect of spectral fluctuations of the RMT eigenvalues, without any dephasing at all.

In conclusion, we have shown that RMT spectral fluctuation effect the probability distribution function $P(G^{\max})$, leading to non-negligible deviations in measurable quantities in the regime $0.1\Delta < k_B T < 0.8\Delta$. In particular, the weak-localization correction α, which was recently used as a probe of dephasing in closed quantum dots, is affected: α is different from 1/4 and, moreover, turns out to be temperature dependent even in the absence of dephasing. At low temperatures, α is reduced to $\alpha \approx 0.2$, which can be below the lower limit of a picket-fence model *with* dephasing. This should also be taken into account while analyzing the ongoing experiments aimed at measuring dephasing times in closed dots in the low-temperature regime $k_B T \lesssim \Delta$. Finally, we would like to note that during the completion of the Brief Report some of the results have been independently arrived at in Ref. 15.

We acknowledge helpful comments from H. Baranger and G. Usaj. This work was supported by ARO, DARPA, and the Alexander von Humboldt foundation.

[1] Y. Alhassid, Rev. Mod. Phys. **72**, 895 (2000).
[2] I.L. Aleiner, P.W. Brouwer, and L.I. Glazman, Phys. Rep. **358**, 309 (2002).
[3] M.A. Kastner, Rev. Mod. Phys. **64**, 849 (1992); L. P. Kouwenhoven, C. M. Marcus, P. L. Mceuen, S. Tarucha, R. M. Westervelt, and N. S. Wingreen, in *Mesoscopic Electron Transport*, edited by L. L. Sohn, L. P. Kouwenhoven, and G. Schoen (Kluwer, Dordrecht, 1997).
[4] A. Jalabert, A.D. Stone, and Y. Alhassid, Phys. Rev. Lett. **68**, 3468 (1992).
[5] A.M. Chang, H.U. Baranger, L.N. Pfeiffer, K.W. West, and T.Y. Chang, Phys. Rev. Lett. **76**, 1695 (1996); J.A. Folk, S.R. Patel, S.F. Godijn, A.G. Huibers, S.M. Cronenwett, and C.M. Marcus, Phys. Rev. Lett. **76**, 1699 (1996).
[6] S.R. Patel, D.R. Stewart, C.M. Marcus, M. Gökçedag, Y. Alhassid, A.D. Stone, C.I. Duruöz, and J.S. Harris, Jr., Phys. Rev. Lett. **81**, 5900 (1998).
[7] Y. Alhassid, Phys. Rev. B **58**, 13 383 (1998).
[8] J.A. Folk, C.M. Marcus, and J.S. Harris, Jr., Phys. Rev. Lett. **87**, 206802 (2001).
[9] A.G. Huibers, M. Switkes, C.M. Marcus, K. Campman, and A.C. Gossard, Phys. Rev. Lett. **81**, 200 (1998); A.G. Huibers, J.A. Folk, S.R. Patel, C.M. Marcus, C.I. Duruöz, and J.S. Harris, Jr., **83**, 5090 (1999).
[10] C.W.J. Beenakker, H. Schomerus, and P.G. Silvestrov, Phys. Rev. B **64**, 033307 (2001).
[11] E. Eisenberg, K. Held, and B.L. Altshuler, Phys. Rev. Lett. **88**, 136801 (2002).
[12] See Y. Alhassid, M. Gökçedağ, and A.D. Stone, Phys. Rev. B **58**, 7524 (1998); Full RMT level fluctuations have also been taken into account in the quadrature approximation by R.O. Vallejos, C.H. Lewenkopf, and E.R. Mucciolo, *ibid.* **60**, 13 682 (1999).
[13] C.W.J. Beenakker, Phys. Rev. B **44**, 1646 (1991).
[14] G. Usaj and H. Baranger (private communication).
[15] T. Rupp, Y. Alhassid, and S. Malhotra, Phys. Rev. B **65**, 193304 (2002).