100 years since Einstein's less known revolution:

From the pollen dance to atoms and back

By David Andelman and Haim Diamant

Abstract

Twentieth century physics was based on three conceptual revolutions, two of which are well known to the general public: the theory of relativity and quantum mechanics. The third revolution – statistical physics – has had less successful public relations despite its vast implications for our daily life. Statistical physics is the theory that allows us to relate the properties of multi-component systems (e.g., the paper or computer display you are currently looking at) to their microscopic components and interactions with the environment. Albert Einstein made crucial contributions to each of these three revolutions, all published during the miraculous year of 1905. In this article we focus on his contribution to statistical physics and its far-reaching impact on a surprisingly broad range of contemporary scientific areas.

The Third Revolution

When we look at a glass of water or a piece of metal, without realizing it we are observing an enormous number of atoms and molecules. In one liter of water, for example, there are about 10^{25} molecules. It is customary to define a quantity called *Avogadro's number* (named after the Italian chemist Amedeo Avogadro) as the number of molecules in a sample of material whose weight in grams is equal to the molecular weight of the material (for instance, the number of molecules in 18 grams of water). Since this number is so large ($N_{\mathrm{A}} = 6.02\times10^{23}$) we can usually think of macroscopic systems as a *continuum*, which is not made up of basic discrete units. On the other hand, when we observe the microscopic world at length scales of a *nanometer* or less, the atomistic description is necessary because atoms and small molecules are about a tenth of a nanometer in size. At this level the world is actually described in terms of discrete units – atoms – and not as a continuum.

Statistical physics is the theory behind our understanding why materials (that is, this enormous and complex collection of discrete particles) are found in different states or phases; how they transform from one phase to another; what determines their elastic, thermal, electric or magnetic properties; and how factors such as temperature and pressure affect all of these. For example, statistical physics (along with quantum mechanics) plays a crucial role in the explanation of electrical properties of solids, and thus in the development of semi-conductors and the computer revolution that followed. It is amazing that Albert Einstein when he was only 26 made decisive

contributions to each of the three revolutions of modern physics, and that each of his three contributions was published in the same year - the Annus Mirabilis, 1905.

Conflict between Two Approaches

In order to properly appreciate the circumstances in which Einstein's 1905 paper on Brownian motion appeared, we should understand that at the start of the 20th century it was not yet self-evident that one could describe materials as consisting of atoms or molecules. On the contrary, there was a passionate controversy between two schools of thought.

On one side stood one of the crowning achievements of 19th century physics – thermodynamics – which supplied an exact description of the behavior of macroscopic materials by treating them as a *continuum* and not as a collection of discrete particles. Thermodynamics is a deterministic theory, which can foretell with certainty how a material will respond to changes of temperature or pressure, when it will undergo a phase transition, and so on. The predictions of thermodynamics were confirmed with great precision by many experiments in the course of the 19th century.

On the other side were the works of the Austrian Ludwig Boltzmann and the British James Clerk Maxwell – the fathers of statistical physics and among the most important scientists of the 19th century. In their works on "*The Molecular Kinetic Theory of Heat*" – as statistical physics was then called – they claimed that the macroscopic description of materials may be derived as *statistical* results of the movement and interrelationships of a huge number of discrete particles, a number of the order of Avogadro's number. This theory is not deterministic, and regards the measured properties of materials, such as pressure or density, as *statistical* quantities with a mean, standard deviation, etc. As early as the 19th century there was mounting and convincing evidence, particularly in chemistry, that all materials are composed of discrete particles, molecules, and that these are composed of even more basic particles, atoms. Still a significant number of scientists at the end of the 19th and start of the 20th century, led by the Austrian physicist Ernst Mach and the German chemist Wilhelm Ostwald, clung to the continuum theory and to thermodynamics as the only correct approach to a description of the behavior of macroscopic materials. In their view, not only was it completely impossible to observe atoms, but also since in the description of material behavior we are only interested in macroscopic characteristics (such as volume, pressure, density) there is no need or point at all to an atomic theory. The French physicist Jean Perrin, who was responsible for the experimental verification of Einstein's predictions, later remarked in response to this: "I find it very difficult to understand this point of view, since what is inaccessible today may become accessible tomorrow… and also because coherent assumptions on what is still invisible may increase our understanding of the visible."

Today we know that the two theories do not conflict and that the laws of thermodynamics are simply the macroscopic result of the laws of statistical physics. It is interesting to note that relics of this old ambiguity still exist. They may be seen, for example, in programs of study and textbooks of undergraduate physics and chemistry courses.

The verification of the atomic picture of matter, and of the statistical theory that describes it, has a history of nearly 200 years. The story begins in a surprising place – neither physics nor chemistry, but rather in botany, 80 years before Einstein's work….

Brown's Pollen Dance

At the beginning of the 19th century the science of botany flourished, and many expeditions, particularly British, reached all the corners of the earth and returned with an enormous range of new species of plants. In 1827 Robert Brown, a globally renowned British botanist, looked through his microscope at a suspension of pollen in water. To his surprise the pollen grains were not at rest but moved without cease in a random, infinite dance. At first Brown thought that the grains were alive. However, with skill and care he attempted to observe particles of similar size (a thousandth of a millimeter – a micrometer) made of inorganic material such as dust or soot. These particles too danced at random. It was clear therefore that this movement, henceforward called *Brownian motion,* was a physical rather than biological phenomenon.

Brownian motion remained without sufficient explanation for several decades. In 1877 the Belgian scientist Joseph Delsaux was first to put forward the explanation that it is related to the discrete particle character of the liquid that surrounds the grains, that is, to the thermal motion of the water molecules and their collisions with the grains. The first to attempt thoroughly to test this idea was the French physicist Louis Gouy in 1888. Gouy showed that the rate of Brownian motion is inversely proportional to the viscosity of the liquid in which the grains are suspended. However, it was Albert Einstein who first formulated a complete theory of the phenomenon. Not only that, Einstein understood its importance as an experimental touchstone, measurable with a simple microscope, for the validity of statistical physics.

The microscopic world and the mesoscopic world

It is important to bear in mind the relevant dimensions. Brown observed particles the size of micrometers using an optical microscope. Micrometer-sized particles are 10,000 times as large as atoms; it is of course impossible to see atoms through an optical microscope. Micron and sub-micron particles, down to single nanometers, belong to the mesoscopic world – a world that contains hundreds of billions (but not Avogadro's number) of atoms and molecules. Nanoscopic and mesoscopic physics today are at the forefront of scientific research. Colloidal scientist Wolfgang Ostwald (son of Wilhelm Otswald) said as long as 100 years ago that "the mesoscopic world is the world of neglected dimensions."

Einstein's Paper on Brownian Motion

In 1905, the same year in which Einstein published his well known papers on relativity theory and on the quantum theory of light (the photoelectric effect), another

paper of his appeared titled: "*On the Movement of Small Particles Suspended in a Stationary Liquid Demanded by the Molecular Kinetic Theory of Heat*". At the beginning of the paper Einstein remarks with characteristic caution that possibly the motion that he discusses is identical to Brownian motion. But in addition to the explanation of Brownian motion Einstein was completely aware of the real and profound significance of his work: "If the movement discussed here can actually be observed (together with the laws relating to it that one would expect to find), then classical thermodynamics can no longer be looked upon as applicable with precision to bodies even of dimensions [as large as those] distinguishable in a microscope: an exact determination of actual atomic dimensions is then possible. On the other hand, had the prediction of this movement proved to be incorrect, a weighty argument would be provided against the molecular-kinetic conception of heat."

Inasmuch as it was not possible to see atoms in Einstein's day – they are simply too small and too speedy – could it be possible to infer their existence by means of the laws of statistical physics for particles that could be seen? Einstein assumed that a Brownian particle in a liquid behaves like a very big atom in equilibrium with the surrounding liquid. The liquid can be thought of as a collection of much smaller particles, moving at random and colliding constantly with each other as well as with the larger particle. The large particle moves with Brownian or diffusive motion within the liquid medium as a result of frequent collisions with large groups of particles of the liquid. Diffusive processes in continuous media (as for example with a solution of water and salt) were known to scientists in the 19th century. Einstein showed in his paper that it was possible to demonstrate a precise connection between such diffusive processes on the macroscopic scale and random processes of discrete particles on the microscopic scale (see box).

Random Walks and Brownian Motion

In order to explain Brownian motion envision a drunken man staggering right and left along a straight line. At each time period Δt *he decides to take one step of length* a *to the right or left at random. The question is: what is the characteristic distance the man has staggered after* N *steps? With each step the location of the drunk changes by* $\Delta x = \pm a$ *. After* N *steps his location will be:* $X = \Delta x_1 + \Delta x_2 + ... + \Delta x_N$ *, but this is a statistical number, which can vary between* $+Na$ *and* $-Na$ *. When the number of steps* N *is very large, the average distance written* $\langle X \rangle$ *will be zero, because neither right nor left is a preferred direction. From the statistical average of the square of the sum*

$$\left\langle X^2 \right\rangle = \left\langle \left(\Delta x_1 + \Delta x_2 + ... + \Delta x_N \right)^2 \right\rangle \approx Na^2$$

we obtain the standard deviation, which is a statistical measure of the characteristic distance the drunk has passed after N *steps of time,*

$$X_0 = \sqrt{\left\langle X^2 \right\rangle} \approx \sqrt{N}a$$

This is an important result. It means that the distance in a random process such as diffusion grows only in proportion to the square root of the number of steps, or in other words by the square root of the time. Despite the considerable simplification, the fact is that particles undergoing Brownian motion in liquid (see figure 1) behave exactly as the random walker. In order to illustrate the difference between diffusion and motion at a constant speed (see figure 2), we take a micrometer-sized particle. After a million steps, each one micrometer in length, the characteristic distance such a particle will move in Brownian motion will be $X_0 = \sqrt{10^6} \times 10^{-6}\,\mathrm{m} = 1\ \mathrm{mm}$ *, only one millimeter, whereas a particle moving at a constant velocity for a million steps in the same direction will advance a distance of* $10^6 \times 10^{-6}\,\mathrm{m} = 1\,\mathrm{m}$ *, one meter, that is, a distance a thousand times longer*

Figure 1. Example of a random Brownian walk of 100 steps in the plane, drawn from a computer simulation. The blue arrow connects the initial point to the final point. Note how short the arrow is relative to the length of the entire path. If the length of a single step is a, the entire path length is $100a$. In contrast to this, if we were to draw many processes such as that of the figure and work out the average arrow length, we would find a length of only $10a$.

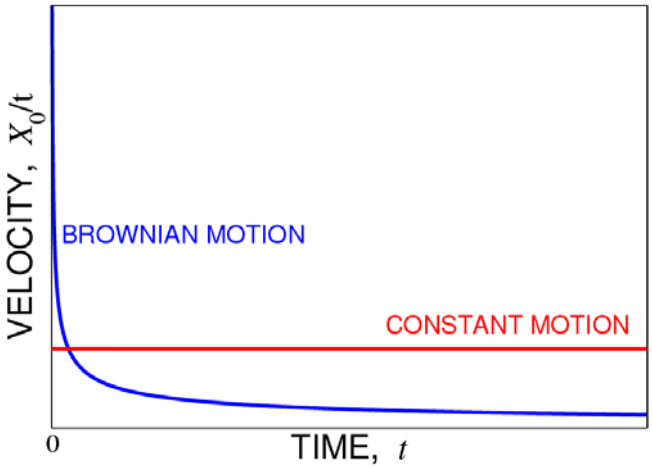

Figure 2. The average velocity of a particle is the ratio between the distance X_0 and the time t required to travel this distance. In the figure the red line shows constant velocity as a function of time. For a Brownian particle, however, the distance grows as $\sqrt{t}$, and so we find an average velocity proportional to $1/\sqrt{t}$, as shown by the blue curve. This means that as we measure average velocity over smaller and smaller time periods, the velocity will grow enormously. This experimental fact had confounded investigators of the Brownian motion before Einstein's theory was published.

Einstein showed that a micrometric particle buffeted at random by particles in the surrounding liquid will fulfill a statistical law according to which the distance the particle passes will increase only as the square root of the time, $X_0 = \sqrt{6Dt}$ (see box). The coefficient in this relation, *D*, is called the *diffusion coefficient*. For a one micron particle suspended in water, $D \approx 4\times10^{-9}\,\mathrm{cm}^2/\mathrm{sec}$. This means that in one second the particle advances a characteristic distance of $X_0 = \sqrt{6Dt} \approx 1.6$ micron – a distance similar to its diameter.

But how to calculate the diffusion constant of the particle? Here too Einstein made a decisive contribution, when he demonstrated that the diffusion constant depends on the relationship between the temperature *T* and the coefficient of friction between particle and liquid, γ:

$$D = \frac{R}{N_A}\frac{T}{\gamma} \propto \frac{T}{\gamma}$$

This elegant result is to this day known as the *Einstein relation.* The coefficient of friction γ depends on the size of the Brownian particle and on the viscosity coefficient of the liquid, but surprisingly the velocity of the particle does not appear in this relation. The coefficient in the Einstein relation contains two constants: the gas constant *R* and Avogadro's number N_A. The Einstein relation therefore provided a

direct method of measuring Avogadro's number by means of observations of the motion of Brownian particles, as will be shown in the next section.

Perrin's Experimental Verification of the Atomic Picture

Since at the start of the 20th century it was not possible to "see" atoms and molecules, scientists had to infer their existence from physical phenomena, which could be measured at macroscopic or mesoscopic length scales, but whose explanation was inextricably connected to "atomic reality" – that is, to the huge number of discrete basic units that make up matter.

Even before Einstein there were experimental estimates of Avogadro's number. But after Einstein's work had appeared in 1905 a number of experiments were carried out, particularly between 1908-1911, in which N_A was measured to great precision. Of these experiments the most noteworthy were those of French scientist Jean Perrin. These proved to be the ultimate experimental verification of Einstein's theory of Brownian motion, as well as proof of the existence of atoms and molecules (see box). Perrin received the 1926 Nobel Prize in physics for this work.

As Brownian motion cannot be directly measured at atomic scales, Perrin used a suspension of powder grains produced from the gum (gamboge) tree, spherical in shape and of micrometric size. Such a suspension can be produced in a liquid and the grains are then magnified by an optical microscope for observation. Perrin and his colleagues performed a precise analysis of the particle trajectories, and thus succeeded in verifying experimentally the ½ power law of Brownian motion – that is, the fact that the characteristic distance a particle goes through grows in direct proportion to the square root of the time.

In addition – this was the crowning achievement of these revolutionary experiments – Perrin succeeded in obtaining an accurate estimate of Avogadro's number, N_A, which was in amazing accordance with estimates that had been made from completely different physical phenomena: thermal radiation of heated bodies, Rayleigh scattering of sunlight by the atmosphere (the effect that makes the sky look blue), and radioactivity.

By 1909 these results had brought even the greatest skeptics to accept the validity of the atomic picture, and put to an end to one of the most loaded disputes in the history of physics.

Measurement of Avogadro's Number

The Einstein relation provides a method of directly measuring Avogadro's number, and that is what Perrin did. The diffusion constant D is calculated from the measurement of the relationship between the distance X_0 that the Brownian particle passes, and the duration of its motion t. The friction constant γ of a spherical particle, with known diameter and moving in a liquid of known viscosity, may be calculated by use of hydrodynamic theory (Stokes' formula). The gas constant R was known from thermodynamic measurements.

Therefore it is plain to see that at a given temperature T Avogadro's number may be calculated by plugging in all the other quantities to the Einstein relation. In 1908 Perrin found a value of 6.4×10^{23} for Avogadro's number, and by 1914 the experimental value matched today's value of 6.022×10^{23} with a four-digit precision.

Before moving on to the far-reaching implications of Einstein's paper on contemporary research let us sum up its three central contributions:

(*) A conclusive explanation of Brownian motion. The explanation was based on analysis of random processes, and showed that the distance passed by a Brownian particle grows with the square root of the time. This revolutionary description of random motion paved the way for an entire field of scientific research in analysis of stochastic processes and signals.

(*) Verification of the atomic picture. Einstein's model required treatment of the liquid surrounding the particle not as a continuous medium but as made up of molecules. What is more, Einstein showed that Avogadro's number can be directly measured from characteristics of Brownian motion.

(*) The relationship between random motion and friction. In his paper Einstein proved the existence of a deep relationship (the Einstein relation) between the diffusion constant and the temperature and friction coefficient. This relationship between random motion (a microscopic phenomenon) and friction (a macroscopic characteristic of matter) laid the groundwork for generalization and applications in many systems, and became one of the cornerstones of statistical physics.

100 Years After Einstein: Implications and Current Applications

Although one hundred years have gone by since Einstein and his colleagues did this work, random processes are still being investigated intensively in a surprising variety of areas of science and technology, and not only in the natural sciences. As examples we have chosen to summarize here a number of applications.

Polymers: Polymers are long flexible molecules that are found in nature as polysaccharides, DNA, proteins and more. Since the start of the 20th century polymers that are by-products of the petrochemical industry have been the basis of all the plastic products so characteristic of our times. Every polymer is made up of base-units (monomers) chemically joined. When a polymer molecule is dissolved in liquid, it is characterized by great flexibility, and any spatial conformation it forms appears to be a *random walk* (figure 3). This walk resembles Brownian motion, but differs from it in one important characteristic: the walk cannot intersect itself ("self-avoiding walk"), since two different monomers cannot occupy the same space.

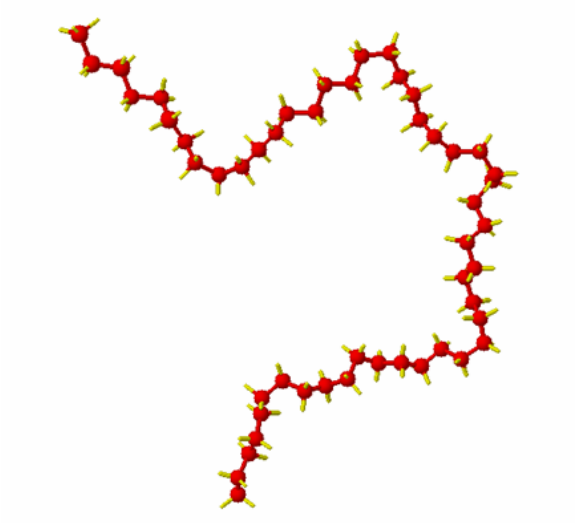

Figure 3. A schematic description of a polyethylene molecule containing 50 base-units (monomers). Such polymers may contain a far larger number of monomers. When they are dissolved in a dilute solution, the statistics of their possible conformations corresponds to that of a self-avoiding random walk.

Statistical analysis of "walks" of polymer chains teaches a great deal about polymers, both for single chains and for polymer material consisting of a great number of chains. We point out two important characteristics:

(*) As a polymer chain can be very long (thousands and even hundreds of thousands of base-units) and flexible, the enormous variety of possible chain conformations (its entropy) dictates to a large extent its material properties. This quality distinguishes polymers from other materials. For instance, we are used to the fact that solid material such as metal becomes less rigid when heated, and so a metal spring under a load, for example, will lengthen when heated (see figure 4a). On the other hand, if we replace the metal spring with a rubber band made of polymeric material, to our surprise we find that the rubber band shrinks when heated (figure 4b) because the higher temperature makes it harder to restrict the polymer conformations (reduce its entropy) by stretching.

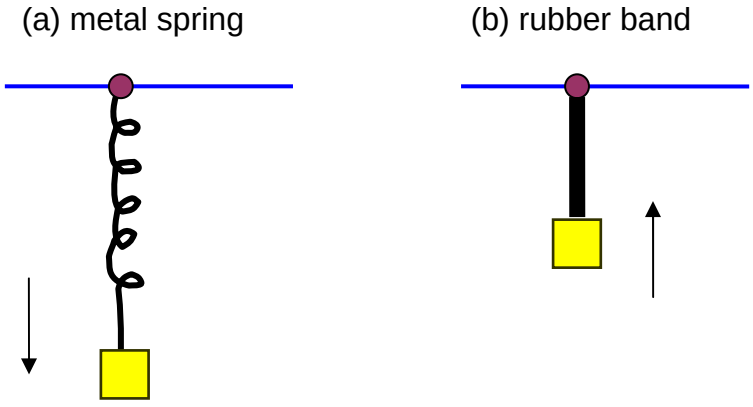

Figure 4. (a) Metal spring under a load. When heated the rigidity of the spring lessens and the weight causes it to lengthen. (b) Polymer rubber band under the identical stress. When heated it becomes rigid and contracts.

(*) As shown in figure 5, a polymer chain that does not self-intersect has a more "swollen" size than that of a Brownian walker of the same contour length. This characteristic of self-avoidance has many implications, which change polymer behavior as opposed to regular random walks.

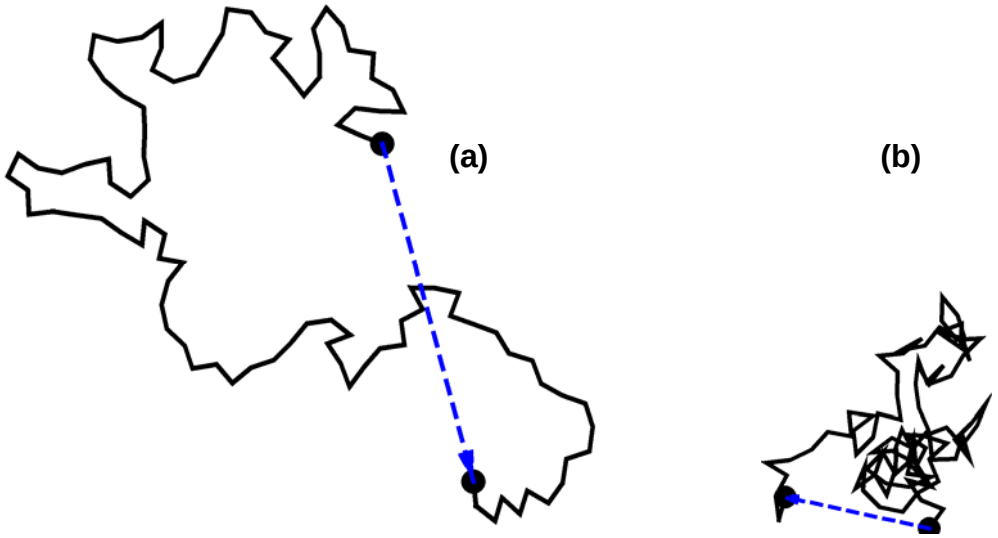

Figure 5. (a) Example of a self-avoiding random walk of 100 steps, drawn from a computer simulation. The statistics of such processes describes the different conformations of polymer molecules in solution (figure 3). The blue arrow joins the starting point to the end point. Note once again how short the arrow is relative to the entire path length. (b) The normal Brownian process of figure 1 is presented alongside the self-avoiding walk, at the same scale. Comparison of the two walks clearly demonstrates how self-avoidance considerably "swells up" the space covered by the random walk.

Anomalous Random Processes: Einstein's work on Brownian motion opened up an entire new area of probability theory, which involves stochastic (random) processes. In the past few decades much research in this area focuses on stochastic processes, which give rise to anomalous diffusion, that is, distances that grow at a greater pace (super-diffusion) or a slower pace (sub-diffusion) than the square root of the time. Super-diffusion can be seen, for example, when the steps of the drunkard's walk are not uniform but may be very large. Such random processes, called *Lévy flights* (named after the French engineer and mathematician Paul-Pierre Lévy) have been observed for example in the way that various birds fly in their search for food. Other examples of anomalous random processes have been found in a variety of phenomena such as weather fluctuations, water permeation through rocks, and heartbeat irregularities.

"To See One Molecule": Einstein and Perrin were obliged to deduce the existence and motion of single molecules indirectly through the motion of a particle 10,000 times as large. In the past two decades science has made a huge leap in the field of microscopy, and today it is possible to directly observe molecules and even single atoms. Experiments at the level of a single molecule are at the forefront of today's research in chemistry, physics, materials science and biology. There are various methods of observing processes at a molecular level. One of these is to chemically attach a "marker" to a molecule to make it easier to see (for example, a fluorescent tag marker).

Figure 6 shows results of recent research at Nagoya University in Japan on the motion of one of the lipid molecules, which make up the membrane of a living cell. This experiment may be regarded as a repeat of Perrin's experiment but this time it is the progress of one molecule in a cell membrane. If this figure is compared to figure 1 of the Brownian walker, both similarity and difference are evident. For a period of several milliseconds the molecule goes through a Brownian walk in all respects, just as Einstein described. However, it turns out that a cell membrane is a more complicated medium than the uniform fluid through which Perrin's and Brown's grains floated. The simple Brownian motion of the molecule is limited to a region with a diameter of a few tenths of a micrometer. After a few milliseconds the molecule can "skip over" to a nearby region and there continue the Brownian motion.

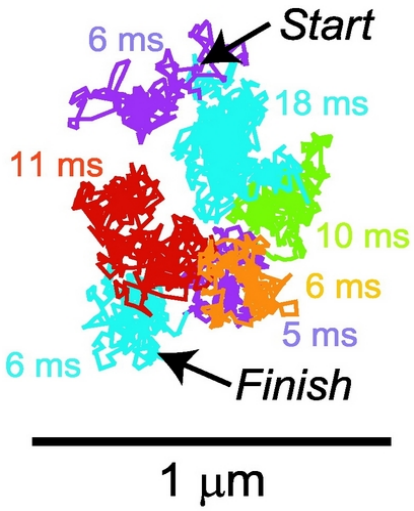

Figure 6. Random walk of one lipid molecule in a cell membrane. The molecule executes Brownian planar motion in a delimited domain for a few milliseconds, then passes on to a nearby domain of the cell membrane. The colors show the different domains. The horizontal line shows a length of one micrometer. The experiment was performed by A. Kusumi and colleagues at Nagoya University. Used by permission of the Annual Review of Biophysics and Biomolecular Structure (2005).

"Noisy Life": Biology is one of the areas in which the importance of fluctuations and random processes is particularly significant. Living processes require operation of defined mechanical and chemical tasks on one hand, and on the other hand flexibility, dynamism and ability for fine adjustment and error correction. Biological systems therefore face a dilemma with regard to fluctuations and random noise. They must be robust enough so that their operation will not be disturbed by extreme disorder. At the same time they must be capable of exploiting the disorder for their own purposes, for example in order to transfer various molecules in and out of the cell. As a result the energies of interactions in biology are not significantly larger than that of the thermal noise. What is more, biological tissue is not rigid as metal or cement… One can therefore say that life processes take place on the verge of thermal noise. For instance electric currents in live cells (brain, heart, etc.) are carried by means of diffusion (that is to say random processes) of ions in an electric field through selective ionic channels located in the cell membranes. It is thus clear that since the appearance of Einstein's

paper we have gradually developed an understanding of how characteristics and functions are defined with relation to microscopic random processes, and this is of central importance for our understanding of the fundamental processes of nature.

An illuminating example from recent times is in regard to enzyme action – the impressive efficiency increase of a biochemical process by means of the appropriate molecule (enzyme). Experiments in enzyme activity have been performed for dozens of years. These are macroscopic experiments, which teach us about the *average* activity of an enormous number of enzyme molecules. They are analogous for our purposes to an experiment, which observes diffusion of a huge number of particles in a macroscopic suspension. In recent years, as we have noted, scientists have achieved the ability to observe a single molecule, and in this case the activity of a single enzyme molecule. Such experiments, by the same analogy, are equivalent to Perrin or Brown observing one single grain. It turns out that a single enzyme molecule is located in an extremely noisy environment, and its chemical activity takes place in a sort of random process of on/off switching.

Another example that has been the center of much recent scientific effort is molecular engines – molecular clusters, which perform various mechanical operations necessary for the functioning of the living cell. On one hand, these are indeed "engines" in the sense that they use "fuel" (chemical energy in the form of ATP molecules) in order to perform mechanical work. On the other hand, unlike macroscopic engines, they operate in an environment full of random fluctuations. Moreover, it seems that they even *exploit* the noise, by inserting asymmetry into the random process, in order to perform their operation more efficiently.

Random Processes in Economics and Communication: Numerous phenomena in our environment involve statistical fluctuations reminiscent of random processes. Particular examples of this are fluctuations of currency rates and stock prices as well as various economic indices; also weather changes over various periods of time. Figure 7 shows changes in the Dow Jones index of leading stocks in the American stock market. The "time step" arbitrarily chosen here is one month, and the drawing shows the "random walk" of stock values from one month to another, beginning from September 1st, 1999 and up to September 1st, 2005. The similarity to the drunkard's random walk discussed earlier is plain. The drunkard's step to the right is equivalent to a rise in the index, and a step to the left – to a drop (or vice versa).

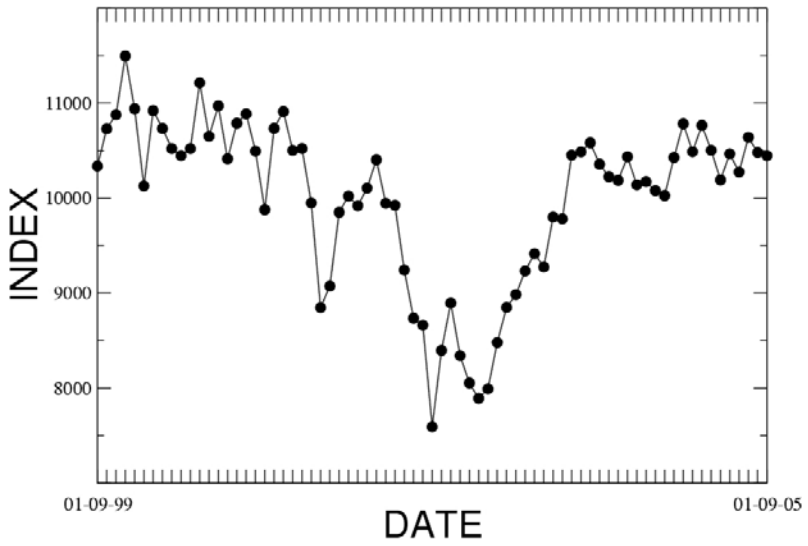

Figure 7. "Random Walk" of the Dow Jones index. Each point represents the index value at the beginning of the month, starting from September 1st 1999 and up until September 1st 2005. Financial analysis and investment groups today use a simulation technique called *Brownian Dynamics* to analyze such "walks".

Obviously processes such as that of the Dow Jones index are far more complex than simple Brownian motion and far from being completely random. Nevertheless, as early as the beginning of the 20th century, five years before the appearance of Einstein's paper, French scientist Louis Bachelier pointed out the possibility of analyzing rate changes in the financial market as random processes. Seventy years later the Americans Fischer Black, Myron Scholes and Robert Merton presented a model analyzing stock rates as a random walk called *Geometric Brownian Motion.* This model has found many applications; in 1997 (after the death of Black) it brought Scholes and Merton the 1997 Nobel Prize in Economics. Investment companies and financial analysts of recent years have increasingly been using analytical tools inspired by the motion of Brown's particles, and based on a technique of computer simulation called *Brownian Dynamics.*

The theory of random motion and signaling has many technological applications as well. A relevant example from the world of optical communications is a phenomenon called PMD (Polarization Mode Dispersion). When a signal advances through a fiber optic, it accumulates distortion as a result of unavoidable slight defects in the fiber structure. This buildup of distortion is a random process, which obeys the same mathematical laws Einstein formulated for Brownian motion.

Epilogue

In his revolutionary article of 1905 Einstein showed how microscopic processes (motion of molecules in a fluid) can be deduced from mesoscopic processes (motion of a grain of pollen). Thus he resolved the controversy concerning the correctness of the atomic picture and the validity of statistical physics. In recent decades we have witnessed more and more research in the opposite direction, "back to the pollen", so

to speak: how to use our increasing knowledge of random processes at the molecular level to draw conclusions and make predictions about the behavior of large complex systems such as the living cell, the organism, the atmosphere, or the stock market.

Although Einstein's third revolution is less well known to the public, it had and still has far-reaching implications for science and for many applications in everyday life. Therefore, from a perspective of a hundred years later, we can say this revolution actually had a wider influence than the other two revolutions Einstein created. With time it has brought forth a new world view in which randomness has a central role. We conclude with a quotation on a similar theme from Mark Haw's article that appeared in January 2005 in Physics World: "Brownian motion was just a slower, subtler revolution: not a headlong charge, but more of a random walk into a vast and unsuspected future."

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Further Reading

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Java realization showing a Brownian particle in a medium of small particles colliding with it constantly. The number of colliding particles and their relative size may be adjusted.

6. http://galileo.phys.virginia.edu/classes/109N/more_stuff/Applets/brownian/brownian.html
Another Java realization.

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Nobel physics prize site for 1926. Includes Perrin's biography and his Nobel lecture summarizing the proof of the existence of the atomic world.

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