ON THE THEORY OF TRIPOD AMPHIPHILES, CHIRAL DISCRIMINATION AND PHASE TRANSITIONS IN LANGMUIR MONOLAYERS

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Chiral discrimination is investigated theoretically for chiral molecules that form an insoluble Langmuir monolayer on the water/air interface. For a specific type of model molecules – tripod amphiphile – the chiral discrimination is calculated for various types of intermolecular interactions: van der Waals, hydrogen bonding, dipoles, charges, etc. In particular, we find that van der Waals interactions prefer chiral segregation whereas electrostatic interactions prefer heterochiral compounds. Monolayer phase diagrams are calculated for both cases.

Chiral molecules are of fundamental importance in biology, organic chemistry and in the design of novel optically active materials [1-3]. Since the origin of *chiral discrimination* and *chiral specificity* is only poorly understood in bulk three-dimensional systems, we restrict ourselves in the present study to a *two-dimensional* geometry. Such a simplified geometry can be realized in a Langmuir monolayer arrangement, where chiral amphiphiles are restricted to lie on the water/air interface. Although relatively few studies on mixtures of chiral monolayers (*racemic mixtures*) presently exist [4-6], the monolayer geometry offers several advantages: (i) it is a simple model for a biological cell membrane with chiral phospholipids. (ii) Chiral interactions are simpler to study in the two-dimensional geometry. (iii) The surface pressure is a convenient thermodynamic field coupled to the area per molecule.

The basis of our analysis [7, 8] is a model chiral molecule which we called a *tripod amphiphile*. The molecule has one chiral center connected to four groups: A, B, C and D. Three of the groups, say, A, B and C are restricted to lie on the water/air interface, while the fourth one, D, is a hydrophobic tail pointing into the air. The fourth group ensures that clockwise and counterclockwise arrangements of the three head groups A, B and C belong to two distinct chiral molecules. These two stereomeric arrangements, D and L, are shown in fig. 1, and they form the two different *enantiomers*, D and L.

The origin of the chiral discrimination in the model is a different effective interaction between the D enantiomer with its twin D, as compared with the



Fig. 1. The two stereomeric arrangements, **D** and **L**, of tripod amphiphiles. The groups A, B, and C lie on the water/air interface and are arranged counterclockwise (clockwise) for **D** (**L**), while the group D is a hydrophobic (aliphatic) tail pointing away from the water surface.

interaction with its mirror image, L. Our quantitative analysis is further simplified and is treated within the following assumptions: (i) only "back-toback" arrangements of the two chiral molecules in a close packed geometry are considered. In this way, there are three discrete 60° rotations of each molecule, yielding nine possible relative orientations of the pair. (ii) A further simplification is to neglect all intermolecular interactions except for the shortest range. For each relative orientation, we take into account only the interactions between the two groups (one of each molecule) that are the closest, as shown in fig. 2.

The two-body partition function can now be calculated by averaging with the proper Boltzmann factor over the nine relative orientations of the two mole-



Fig. 2. Top view of one of the possible nine orientations used to calculated the two-body partition functions, Z_{DD} and Z_{DL} .

cules: $Z_{DD} = Z_{LL}$ for a pair of the same enantiomers, and $Z_{LD} = Z_{DL}$ for a pair of opposite enantiomers,

$$Z_{\rm DD} = Z_{\rm LL} = f_{\rm BC}^2 + f_{\rm AB}^2 + f_{\rm CA}^2 + 2f_{\rm CC}f_{\rm AB} + 2f_{\rm BB}f_{\rm CA} + 2f_{\rm AA}f_{\rm BC}$$
(1)

and

$$Z_{\rm LD} = Z_{\rm DL} = f_{\rm AA} f_{\rm BB} + f_{\rm BB} f_{\rm CC} + f_{\rm CC} f_{\rm AA} + 2f_{\rm AB} f_{\rm BC} + 2f_{\rm BC} f_{\rm CA} + 2f_{\rm CA} f_{\rm AB} ,$$
(2)

where $f_{ij} = \exp(-V_{ij}/k_{\rm B}T)$, and V_{ij} is the interaction between the two groups *i*, j = A, B, C of the two molecules; *T* is the temperature and $k_{\rm B}$ is the Boltzmann constant.

The chiral discrimination parameter is conveniently defined as $\Delta = Z_{DD} - Z_{DL}$. From (1) and (2),

$$\Delta = f_{BC}^{2} + f_{AB}^{2} + f_{CA}^{2} - f_{AA}f_{BB} - f_{BB}f_{CC} - f_{CC}f_{AA} + 2f_{CC}f_{AB} + 2f_{BB}f_{CA} + 2f_{AA}f_{BC} - 2f_{AB}f_{BC} - 2f_{BC}f_{CA} - 2f_{CA}f_{AB}.$$
 (3)

 Δ positive indicates a *homochiral* preference and thus a chiral segregation. This is the homochiral (HOC) case, while Δ negative is the *heterochiral* (HEC) case where chiral segregation is disfavored. We discuss now several examples where the chiral discrimination parameter Δ is calculated for specific group-group interactions, V_{ij} , e.g., van der Waals, dipole-dipole, or hydrogen bonding.

1. Free rotating limit. For high temperatures, $V_{ij} \ll k_B T$, yielding $\Delta = 0$ when only first order terms in V_{ij} are included. This is the free rotating limit for which previous studies have shown [9] that two-body interactions do not contribute to the chiral discrimination.

2. van der Waals interactions. When all the intermolecular interactions are dispersion-like (van der Waals),

$$V_{ij} = -M\alpha_i \alpha_j , \qquad (4)$$

where $\alpha_i = \alpha_A$, α_B , α_C is the polarizability of the *i*th group. In this case we have shown [7, 8] that our model always predicts that $\Delta < 0$ (HEC). For example, a high temperature expansion yields

$$\Delta \simeq -\frac{1}{2} \left(\frac{M}{k_{\rm B}T}\right)^3 (\alpha_{\rm A} - \alpha_{\rm B})^2 (\alpha_{\rm B} - \alpha_{\rm C})^2 (\alpha_{\rm C} - \alpha_{\rm A})^2 \le 0.$$
(5)

Similarly, a low temperature expansion for $\alpha_A > \alpha_B$, α_C gives

$$\Delta \simeq -\exp\left(\frac{M\alpha_{\rm A}^2}{k_{\rm B}T}\right) \left[\exp\left(\frac{M\alpha_{\rm B}^2}{k_{\rm B}T}\right) + \exp\left(\frac{M\alpha_{\rm C}^2}{k_{\rm B}T}\right) - 2\exp\left(\frac{M\alpha_{\rm B}\alpha_{\rm C}}{k_{\rm B}T}\right)\right] \le 0.$$
(6)

3. Two van der Waals groups and one charged group. Suppose that group A and B interact purely via van der Waals interaction and C is a charge group. For $f_{CC} \ll 1$ and $f_{AC} = f_{BC} \ll 1$,

$$\Delta \simeq \exp\left(\frac{2M\alpha_{\rm A}\alpha_{\rm B}}{k_{\rm B}T}\right) \left[1 - \exp\frac{M(\alpha_{\rm A} - \alpha_{\rm B})^2}{k_{\rm B}T}\right] \le 0.$$
⁽⁷⁾

4. Electrostatic interactions. Two of the three groups are charged, e.g. B positive and C negative with $w = f_{BC} \gg f_{BB} = f_{CC}$. If the third group is apolar and interacts equivalently with B and C, then

$$\Delta = (w - f_{BB})(w + f_{BB} + 2u - 4v), \qquad (8)$$

where $u = f_{AA}$ and $v = f_{AB} = f_{CA}$. In a simpler case of dominant electrostatic interactions, $w = f_{BC} \ge f_{BB}$, and from (8) we get HOC ($\Delta > 0$) as soon as w > 2(2v - u). One of the main conclusions of our study is the difference betweeen van der Waals interactions which prefer a HEC arrangement and electrostatic interactions (between two oppositely charged groups) which prefer a HOC arrangement.

5. Strongly preferred like groups. For $f_{ij} = 0$ when i and j are not equal, $\Delta < 0$,

$$\Delta = -[f_{AA}f_{BB} + f_{BB}f_{CC} + f_{CC}f_{AA}] < 0.$$
(9)

6. Strongly preferred unlike groups. If the repulsion between like groups is very strong, we can set $f_{ii} = 0$. Calling $u = f_{AB}$, $v = f_{BC}$ and $w = f_{CA}$,

$$\Delta = u^2 + v^2 + w^2 - 2uv - 2vw - 2wu .$$
⁽¹⁰⁾

The boundary $\Delta = 0$ defines a cone of revolution in the (u, v, w) parameter space, centered around the (111) direction. The HEC ($\Delta < 0$) region lies within the cone whereas the HOC region lies outside.

7. One "passive" group. If one of the groups, say A, interacts in an "indifferent way" with all others, $f_{Ai} = w$ for any i = A, B, C,

$$\Delta = (f_{\rm BC} - w)^2 - (f_{\rm BB} - w)(f_{\rm CC} - w).$$
(11)

 Δ can have both signs; in particular, if $f_{\rm BC}$ is larger than any other interactions, then $\Delta > 0$ (HOC).

The chiral discrimination parameter Δ calculated above can be used as an important parameter to describe the thermodynamics of chiral (racemic) mixtures. In a Langmuir trough, the area per molecule is controlled via the surface pressure II. To simulate the variable area per molecule, a symmetric lattice-gas model [8, 10-11] with three components (spin one) can be used: two of the components are the two enantiomers, **D** and **L**, and the third one is a *vacancy* **V**. Because of the chiral symmetry there are only four independent interaction parameters: $E_{LL} = E_{DD}$, $E_{DL} = E_{LD}$, $E_{DV} = E_{LV}$, and E_{VV} . The interaction parameters $E_{ij} = -k_B T \log Z_{ij}$ are calculated from the two-body partition function of the pair (*ij*).

The free energy of the three-component mixture is $F = E - TS^{\text{mix}}$, where E is the internal energy and S^{mix} is the entropy of mixing,

$$S^{\text{mix}} = -k_{\text{B}}(\phi_{\text{D}}\log\phi_{\text{D}} + \phi_{\text{L}}\log\phi_{\text{L}} + \phi_{\text{V}}\log\phi_{\text{V}})$$
(12)

and

$$E = \frac{1}{2}\phi_{\mathbf{D}}^{2}E_{\mathbf{D}\mathbf{D}} + \frac{1}{2}\phi_{\mathbf{L}}^{2}E_{\mathbf{L}\mathbf{L}} + \frac{1}{2}\phi_{\mathbf{V}}^{2}E_{\mathbf{V}\mathbf{V}} + \phi_{\mathbf{D}}\phi_{\mathbf{L}}E_{\mathbf{D}\mathbf{L}} + \phi_{\mathbf{D}}\phi_{\mathbf{V}}E_{\mathbf{D}\mathbf{V}} + \phi_{\mathbf{L}}\phi_{\mathbf{V}}E_{\mathbf{L}\mathbf{V}},$$
(13)

where $\phi_{\rm D} + \phi_{\rm L} + \phi_{\rm V} = 1$ are the three volume (area) fractions of the three components **D**, **L** and **V**, respectively.

The following reduced densities are introduced for convenience:

$$Q = \phi_{\rm L} + \phi_{\rm D} = 1 - \phi_{\rm V} \tag{14}$$

and

$$M = \phi_{\rm D} - \phi_{\rm L} \,. \tag{15}$$

The experimentally controlled D/L mole fraction x is

$$x = \frac{1}{2}(1 - M/Q).$$
(16)

Expressing E and S^{mix} as functions of the new densities Q, M, a common tangent construction of the free energy F reveals any multiphase coexistence

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Fig. 3. Schematic homochiral phase diagram in the surface pressure Π – chiral composition x plane, for a fixed temperature. For more details see ref. [8].

regions. The surface pressure Π can also be calculated from the free energy F. Details are given in ref. [8].

The monolayer phase diagram in the (Π, x) plane, for fixed temperature, is shown in fig. 3 for the HOC ($\Delta > 0$). The model gives a phase diagram similar to the phase diagram for a bulk chiral mixtures [2] with coexistent regions of the disordered liquid with the two solids: the L-solid and the D-solid. For the HEC ($\Delta < 0$) case, a more complicate treatment is needed even in a mean-field level and the expected phase diagram is plotted in fig. 4.

In conclusion, a simple model for chiral discrimination in a monolayer geometry is proposed. For such tripod amphiphiles, the chiral discrimination is calculated from a single parameter and several different types of intergroup interactions, e.g. van der Waals, electrostatic, etc. are considered. The chiral discrimination parameter is used, then, as an important parameter to predict the monolayer phase diagrams for the heterochiral and homochiral cases.



Fig. 4. Schematic heterochiral phase diagram in the surface pressure Π – chiral composition x plane, for a fixed temperature. For more details see ref. [8].

The theoretical ideas presented in this paper have been developed in collaboration with P.G. de Gennes. The support of the US–Israel Binational Science Foundation (under grant No. 87-00338) and the Bat Sheva de Rothschild Foundation is gratefully acknowledged.

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