Lower Critical Dimension of the Random-Field Ising Model: A Monte Carlo Study

David Andelman
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

H. Orland[(a)] and L. C. R. Wijewardhana
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 15 August 1983)

This paper presents extensive Monte Carlo simulations of the random-field Ising model in various dimensions for long times in moderately large systems, and specifically addresses the question of whether the lower critical dimension is 2 or 3. The authors find long-range order for $d=3$ and no long-range order for $d=2$. The marginality of the $d=2$ case is further checked by studying a system in $d=\ln 8/\ln 3 \approx 1.89$ dimensions simulated by a fractal; the authors thus conclude that the lower critical dimension is 2.

PACS numbers: 75.40.Dy, 05.50.+q, 75.10.Hk

Since the paper of Imry and Ma,[1] the question of the lower critical dimension d_l (below which long-range order cannot occur) of the random-field Ising model (RFIM) has been a matter of controversy. Theoretical works can be classified in one of two categories:

(i) The simple domain argument of Imry and Ma, later on refined by including roughening effects,[2] concludes that $d_l = 2$. This is also supported by numerical calculations[3] which specifically simulate the dynamics of interfaces in $d=2$.

(ii) A different approach[4] emphasizing the importance of roughening yields $d_l = 3$. Similarly, the ϵ expansion[5] (which is an expansion around the upper critical dimension, $d_c = 6$) indicates that order by order, the critical behavior of a d dimensional RFIM, in the presence of a Gaussian random field, is identical to that of a nonrandom Ising model in $d-2$ dimensions, thus implying $d_l = 3$ (since for the pure case $d_l = 1$). This argument has been generalized to a nonperturbative treatment[6] by the use of supersymmetries. However, none of these works can convincingly enough resolve the controversy. Indeed all the roughening arguments rely heavily on simplified geometries (e.g., solid-on-solid[3] interfaces) and scaling forms of the surface free energy. Even though the supersymmetry arguments are aesthetically attractive, and certainly correct close to $d_c = 6$, it is not clear whether they can predict correctly the lower critical dimension of the RFIM. In fact, the possibility of multiple solutions of the mean-field equation, as well as the possible change of sign of the Jacobian, is ignored. Also, the validity of a Landau-Ginzburg-Wilson theory is questionable around d_l.

On the experimental side, a similar controversy exists. So far, all experimental works have used a modified version of the Fishman-Aharony prescription[7] where the RFIM is realized by a diluted antiferromagnet in a uniform magnetic field. One series of experiments[8] yields $d_l > 3$, whereas others[9] yield $d_l = 2$. It is to be stressed that these experimental realizations are only approximations to the RFIM. Let us also emphasize that in order to really see long-range ordering, experiments ought to be done in presence of a small symmetry-breaking field; otherwise, if the system is cooled down, domains, induced by the random field, can appear. This is analogous to droplet formation in the liquid-gas transition along the coexistence isotherms. For the diluted antiferromagnetic case, the symmetry-breaking field is a staggered magnetic field which cannot be produced experimentally.

Numerical simulations performed on the RFIM are free of all these difficulties. However, previous Monte Carlo (MC) calculations[10] done on the RFIM with $\pm H$ field distribution cannot, in our opinion, settle this dispute. Indeed, the sample sizes and running times are too small, and most important, the absence of a small symmetry-breaking field may lead to ambiguous conclusions. It thus seems important to perform a detailed computer experiment on the RFIM with Gaussian random field, the advantage being that it is done directly on the system which is studied in most theoretical works and all adjustable parameters can be varied. Thus, we have carried out a complete and thorough study of the RFIM in a Gaussian random field, in various dimensions. We find long-range order in three dimensions; the two-dimensional case is marginal and compatible with no long-range order. The marginality of the $d=2$ case is further checked by making calculations on a fractal lattice which simulates $d=\ln 8/\ln 3\approx 1.89$. For the fractal, we find no long-range order.

Our results were obtained with use of the MC heat-bath method[11] and in addition their consistency was confirmed by the MC Metropolis method.[11] Several sample sizes were used in each dimension with periodic boundary conditions (typically of the order of 2000 spins). The procedure we use in order to achieve thermodynamic equilibrium is a modified field-cooling method. We start at high temperature (where the system is clearly paramagnetic), in the presence of a quenched Gaussian random field H_i of standard deviation H, and of a small constant external field, $H_{\rm ext}$, which we take to be 2% of H. Also, we keep only random-field configurations for which $N^{-1}\sum_{i=1}^{N} H_i \ll H_{\rm ext}$. More specifically, the Hamiltonian for the RFIM is

$$-\beta\mathcal{H}=J\sum_{\langle i,j\rangle} S_i S_j+\sum_i (H_i+H_{\rm ext})S_i .$$

We then cool down the system slowly enough to achieve thermal equilibrium and good statistics, typically 15 000 Monte Carlo steps per spin (MCS) out of which the first 5000 MCS are discarded before taking averages (this has been checked with much longer runs of up to 100 000 MCS). Whenever the system builds up a magnetization ($M > 0.2$), the constant external field is reduced by half at the next temperature in the cooling process in order to check that the magnetization is not forced upon by the external field. This turns out to be particularly important in two dimensions where it is observed that a transient magnetization can build up and flip after the constant field is reduced. Finally, quenched averages of observables have been performed over ten different configurations of the Gaussian random field. We present the curves for the magnetization M, the susceptibility χ, and the specific heat C_v, as a function of temperature. We stress that although in each case we present results for one specific sample size, the sample size has been varied in order to check the consistency of our results. Similarly, we have checked our calculations for various values of H/J. Numerical simulations of disordered systems are subject to two types of errors: (i) For each configuration of the random field, there is the usual stochastic error, due to the finite number of MCS. Except for $d=2$, where thermal fluctuations are particularly important, the number of MCS has been chosen so that this error remains smaller than 2% (close to T_c it can reach 10%). (ii) The average over disorder introduces an additional type of error. Indeed, the small variations of the critical temperature with the random-field configuration induce large errors close to T_c, because of the fact that all measured observables are singular at T_c. We display these error bars in Fig. 1, except for χ very close to the transition. Average observables clearly indicate a sharp transition in $d=3$.

(i) *$d=3$ (Fig. 1)*.—In three dimensions, the situation is fairly clear. The results are reported for $H/J=1$ and lattices of $13\times13\times13$ spins (lattice sizes of $10\times10\times10$ and $15\times15\times15$ were also used with no noticeable difference). In all our samples we find that there is a sharp transition with weaker singularities than in the pure case, at a critical temperature which varies by less than 5% as the random-field configuration is varied (with fixed H/J). This is clearly seen on the magnetization, susceptibility, and specific-heat curves. As the system is cooled down, it always builds up a magnetization in the direction

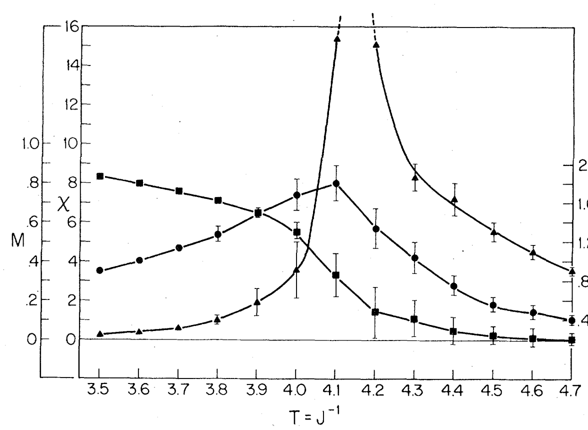

FIG. 1. RFIM in $d=3$: magnetization M (squares), susceptibility χ (triangles), and specific heat C_v (circles) as a function of $T=J^{-1}$. Each data point represents the quenched average over ten Gaussian random-field configurations. Lattices of $13\times13\times13$ spins and running time of 15 000 MCS (of which the first 5 000 MCS are discarded) are used to compute thermal averages. The contribution of the constant external field is subtracted from the magnetization, $M=M_{\rm calc}-\chi H_{\rm ext}$, where $M_{\rm calc}$ is the MC result. $T_c\approx 4.15$ for the RFIM should be compared with $T_c=4.51$ for the pure system.

of H_{ext}, and this magnetization persists when H_{ext} goes to zero. The ground state of the system is completely magnetized with only one domain (this was also observed in a much shorter MC run done by Landau *et al.*[10]). If domains of size larger than our lattice size were caused by the random field, one should expect, for the procedure described above, an equal number of configurations with up and down magnetizations. Since the probability of getting ten consecutive up configurations, just by chance, is 2^{-10} and we never see a magnetization opposite to the external field when the random-field configuration is varied, we conclude that even for much larger systems, formation of domains is highly improbable in the low-temperature phase. We also did five scans of H/J, keeping T constant within each scan ($T=1-5$). We find a critical $H_c/J \approx 2.75$ (to be compared with mean-field value[12] of 4.79) above which there is no magnetization for finite T. For all values of $H/J < H_c/J$, we find a magnetization and a transition with $T_c \to 0$ as $H/J \to H_c/J$. Further numerical work (using finite-size scaling) is currently in progress in order to calculate the corresponding critical exponents.

As another elaborate check on the existence of order in $d=3$, we took one of the RFIM configurations at $T=4.0$ (just below the observed T_c) and $H_{ext}/J=0.01$, and performed a MC run of 10^6 MCS. Observing the time evolution of the M by averaging over every 5 000 consecutive MCS, we found no downward shift in the magnetization as a function of time. The magnetization stabilized around $M=0.610$, with fluctuations of the same order of magnitude, 0.005, throughout the MC run. When $H_{ext}=0$, we checked that overturns due to finite-size effects occur every 2×10^5 MCS, both in the pure and random 13^3 systems just below their respective T_c's.

(ii) *d=2.*—The results for two dimensions are quite different. As the samples are cooled, we observe three types of behavior: In some cases, a magnetization parallel to H_{ext} builds up, similarly to the nonrandom case; however, in half of our samples, either the system builds up a magnetization opposite to H_{ext}, or domains appear, which persist to low temperatures. This should be constrasted with the 45^2 pure system, where overturns due to finite-size effects, just below T_c, occur approximately every 10^5 MCS. In the samples where the system does have a magnetization, the susceptibility and the specific heat exhibit a peak; in the samples with domains, the susceptibility shows a broad plateau (probably due to the trapping in metastable states). In all cases, the system exhibits large fluctuations, and the statistical errors we get with typical samples of 45×45 spins averaged over 15 000 MCS are too large to allow meaningful results. Apparently, the sizes necessary in order to achieve good statistics are prohibitive (we have performed some runs on 60×60 lattices and up to 100 000 MCS with no noticeable improvement on the statistics). However, the fact that H_{ext} cannot always force the magnetization in its direction and the existence of large fluctuations (compared to $d=3$) are evidence that $d=2$ is the lower critical dimension.

(iii) *Fractal* $d=\ln 8/\ln 3 \approx 1.89$.—The marginality $d=2$ is even more apparent when the two-dimensional behavior is compared with that of a spin system in $d<2$. We have constructed noninteger dimensional lattices by using fractals.[13] The fractal lattice we used is constructed in a self-similar way as is shown in Fig. 2. Although we cannot prove that a fractal of Hausdorff dimension d_H represents an analytic continuation of a Euclidian space of dimension d, it is tempting to draw an analogy between the two. For instance, with the fractal we chose, the average connectivity of the lattice is reduced compared to the two-dimensional lattice, and the effective coordination number is $2d=3.78$. We thus claim that this fractal lattice provides some information on the physics in $d<2$. The size of our sample is 81×81, and in the absence of a random field, the system exhibits a ferromagnetic transition at $T_c=2.1$ (compared to $T_c=2.27$ for the square lattice). However, using the cooling procedure described above, we observe that the introduction of very small disorder ($H/J=0.5$) eliminates the transition. In fact, on all samples we tried,

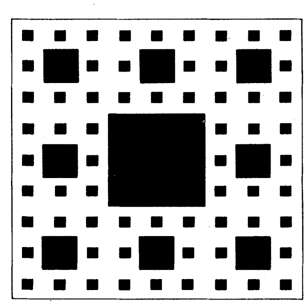

FIG. 2. Fractal lattice with Hausdorff dimension $d_H = \ln 8/\ln 3 \approx 1.89$. The lattice size is 81×81 and spins occupy the blank area only. The sizes of the various black areas are 3×3, 9×9, and 27×27.

the magnetization remains zero down to very low temperatures ($T=0.1$), and the system is made up of many domains of up and down spins. Relaxation times are short enough so that 15 000 MCS is sufficient to provide reasonable statistics.

In conclusion, we summarize our results. (i) In $d=3$, for all samples we checked, the system undergoes a sharp transition to a ferromagnetic state with a magnetization parallel to the (vanishingly) small external field. (ii) In $d=2$, some samples have a magnetization parallel to the external field, and some are opposite. We thus conclude that the system will break up into domains so that there is no ferromagnetic transition (of course, our results do not exclude other types of ordering such as a spin-glass). Also, fluctuations and relaxation times are unusually long (compared to higher- or lower-dimensional systems). (iii) Finally, in $d<2$ ($d\approx 1.89$), weak disorder destroys long-range order. In view of these results, we think that we have presented Monte Carlo evidence that the lower critical dimension of the RFIM is 2.

We thank A. N. Berker, R. J. Birgeneau, E. Brezin, R. Giles, R. Jackiw, S. Mochrie, and J. W. Negele for useful discussions. This work was supported in part by the U. S. Department of Energy under Contract No. DE-AC02-76ERO3069 and in part by the National Science Foundation through Grant No. DMR81-19295.

[(a)]On leave of absence from Service de Physique Theorique, Centre d'Etudes Nucléaires de Saclay, F-91191 Gif-sur-Yvette Cedex, France.

[1]Y. Imry and S.-k. Ma, Phys. Rev. Lett. 35, 1399 (1975).

[2]G. Grinstein and S.-k. Ma, Phys. Rev. Lett. 49, 685 (1982); J. Villain, J. Phys. (Paris), Lett. 43, L551 (1982); A. Aharony and E. Pytte, Phys. Rev. B 27, 5872 (1983).

[3]J. F. Fernandez, G. Grinstein, Y. Imry, and S. Kirkpatrick, Phys. Rev. Lett. 51, 203 (1983).

[4]K. Binder, Y. Imry, and E. Pytte, Phys. Rev. B 24, 6736 (1981); E. Pytte, Y. Imry, and D. Mukamel, Phys. Rev. Lett. 46, 1173 (1981); H. S. Kogon and D. J. Wallace, J. Phys. A 14, L527 (1981).

[5]G. Grinstein, Phys. Rev. Lett. 37, 944 (1976); A. Aharony, Y. Imry, and S.-k. Ma, Phys. Rev. Lett. 37, 1367 (1976); A. P. Young, J. Phys. C 10, L257 (1977).

[6]G. Parisi and N. Sourlas, Phys. Rev. Lett. 43, 744 (1979); J. L. Cardy, unpublished; A. Niemi, Phys. Rev. Lett. 49, 1808 (1982).

[7]S. Fishman and A. Aharony, J. Phys. C 12, L729 (1979).

[8]H. Yoshizawa, R. A. Cowley, G. Shirane, R. J. Birgeneau, H. J. Guggenheim, and H. Ikeda, Phys. Rev. Lett. 48, 438 (1982), and references therein.

[9]D. P. Belanger, A. R. King, and V. Jaccarino, Phys. Rev. Lett. 48, 1050 (1982).

[10]D. P. Landau, H. H. Lee, and W. Kao, J. Appl. Phys. 49, 1358 (1978); I. Morgenstern, K. Binder, and R. M. Hornreich, Phys. Rev. B 23, 287 (1981).

[11]For a review, see K. Binder, in *Monte-Carlo Methods in Statistical Physics*, edited by K. Binder (Springer-Verlag, New York, 1979).

[12]A. Aharony, Phys. Rev. B 18, 3318 (1978).

[13]B. Mandelbrot, *Fractals: Form, Chance, and Dimension* (Freeman, San Francisco, 1977).